

# The physics of sailing

Bryon D. Anderson

Sails and keels, like airplane wings, exploit Bernoulli's principle. Aerodynamic and hydrodynamic insights help designers create faster sailboats.

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**In addition to the recreational pleasure** sailing affords, it involves some interesting physics. Sailing starts with the force of the wind on the sails. Analyzing that interaction yields some results not commonly known to non-sailors. It turns out, for example, that downwind is not the fastest direction for sailing. And there are aerodynamic issues. Sails and keels work by providing "lift" from the fluid passing around them. So optimizing keel and wing shapes involves wing theory.

The resistance experienced by a moving sailboat includes the effects of waves, eddies, and turbulence in the water, and of the vortices produced in air by the sails. To reduce resistance effectively by optimizing hulls, keels, and sails, one has to understand its various components.

## Wind power

Moving air has kinetic energy that can, through its interaction with the sails, be used to propel a sailboat. Like airplane wings, sails exploit Bernoulli's principle. An airplane wing is designed to cause the air moving over its top to move faster than the air moving along its undersurface. That results in lower pressure above the wing than below it. The pressure difference generates the lift provided by the wing.

There is much discussion of whether the pressure difference arises entirely from the Bernoulli effect or partly from the wing's impact and redirection of the air. Classic wing theory attributes all the lift to the Bernoulli effect and ascribes the difference in wind speeds above and below the wing to the wing's asymmetric cross-sectional shape, which caused the path on top to be longer. But it's well known that an up-down symmetrical wing can provide lift simply by moving through the air with an upward tilt, called the angle of attack. Then, despite the wing's symmetry, the wind still experiences a longer path and thus greater speed over the top of the wing than under its bottom. A NASA website has an excellent discussion of the various contributions to lift by an airplane wing.<sup>1</sup> It disputes the conventional simple version of wing theory and emphasizes that lift is produced by the turning of the fluid flow.

The case is similar for sailboats. A sail is almost always curved and presented to the wind at an angle of attack. The situation is shown schematically in figure 1a. The wind moving around the "upper," or downwind, side of the sail is forced to take the longer path. So the presence of the surrounding moving air makes it move faster than the air passing along the "lower," or upwind, side of the sail. Measurements confirm that relative to the air pressure far from the sail, the pressure is higher on the upwind

side and lower on the downwind side.

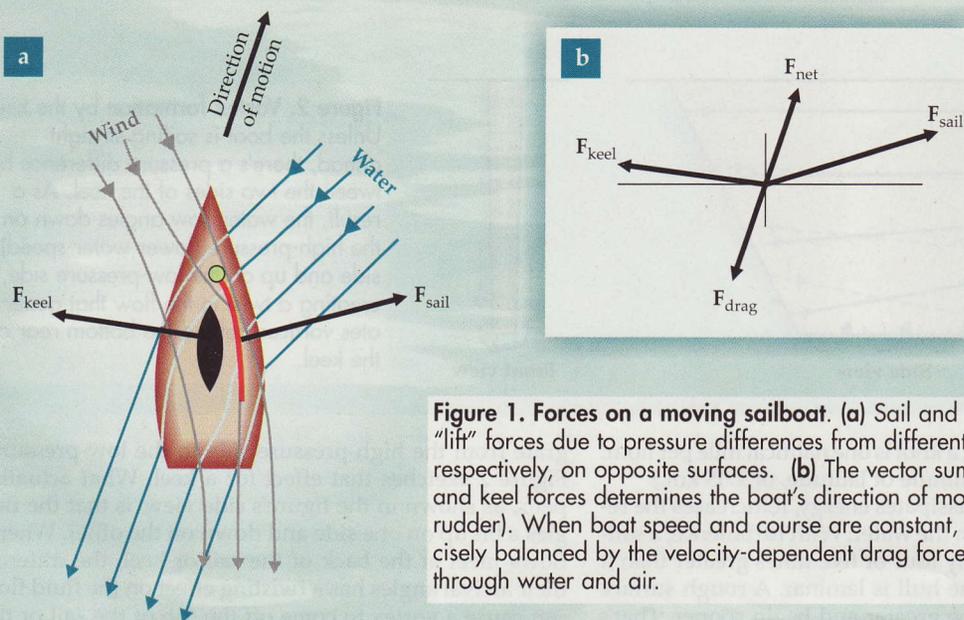
For downwind sailing, with the sail oriented perpendicular to the wind direction, the pressure increase on the upwind side is greater than the pressure decrease on the downwind side. As one turns the boat more and more into the direction from which the wind is coming, those differences reverse, so that with the wind perpendicular to the motion of the boat, the pressure decrease on the downwind side is greater than the pressure increase on the upwind side. For a boat sailing almost directly into the wind, the pressure decrease on the downwind side is much greater than the increase on the upwind side.

Experimenting with what can be done, a beginner finds some surprising results. Sailors know well that the fastest point of sail (the boat's direction of motion with respect to the wind direction) is not directly downwind. Sailboats move fastest when the boat is moving with the wind coming "abeam" (from the side). That's easily understood: When a sailboat is moving directly downwind, it can never move faster than the wind because, at the wind speed, the sails would feel no wind. In fact, a boat going downwind can never attain the wind speed because there's always some resistance to its motion through the water.

But when the boat is moving perpendicular to the wind, the boat's speed doesn't decrease the force of the wind on the sails. One sets the sails at about 45° to the direction of motion—and to the wind. The boat's equilibrium speed is determined by the roughly constant force of the wind in the sails and the resistance against the boat's motion through the water. If the resistance can be made small, the velocity can be large. That's seen most dramatically for sail iceboats, which skate on the ice with very little resistance. They can glide along at speeds in excess of 150 km/h with the wind abeam at speeds of only 50 km/h! Of course sailboats plowing through the water experience much more resistance. Nonetheless, some specially constructed sailboats have attained speeds of more than twice the wind speed.

## Keels

It was recognized centuries ago that a sailboat needs something to help it move in the direction in which it's pointed rather than just drifting downwind. The answer was the keel. Until the development of modern wing theory, it was thought that one needed a long, deep keel to prevent side-slipping. But now it's understood that a keel, like a sail, works by providing sideways lift as the water flows around it, as shown in figure 1a. A keel must be symmetrical for



**Figure 1. Forces on a moving sailboat.** (a) Sail and keel produce horizontal “lift” forces due to pressure differences from different wind and water speeds, respectively, on opposite surfaces. (b) The vector sum of lift forces from sail and keel forces determines the boat’s direction of motion (assuming there’s no rudder). When boat speed and course are constant, the net lift force is precisely balanced by the velocity-dependent drag force on the boat as it plows through water and air.

the sailboat to move to either side of the wind.

A keel works only if the motion of the boat is not exactly in the direction in which it’s pointed. The boat must be moving somewhat sideways. In that “crabbing” motion, the keel moves through the water with an angle of attack. Just as for the sails in the wind, that causes the water on the “high” (more downstream) side of the keel to move faster and create a lower pressure. Again, the net lift force on the keel is due to the combination of that decreased pressure on the high side and increased pressure on the other (low) side.

In figure 1b, the keel lift thus generated points almost in the opposite direction from the lift provided by the sails. The two vectors can be resolved into components along and perpendicular to the boat’s direction of motion. For a sailboat moving in equilibrium—that is, at constant speed in a fixed direction—the transverse lift components from sail and keel cancel each other. The component of the driving force from the sails in the direction of motion is the force that is actually moving the boat forward. For equilibrium motion, that force is balanced by the opposing component of the keel lift plus the total resistive force.

Wing theory, developed over the past 100 years for flight, indicates that the most efficient wing is long and narrow. Vortices produced at the wing tip cost energy. A long, narrow wing maximizes the ratio of lift to vortex dissipation, thus providing the best performance for a given wing surface area. That also applies to sailboat sails and keels.

It is now recognized that the most efficient keels are narrow from front to back and deep. Such a keel can have much less surface area than the old long keels. Less area means less resistance. Most modern racing sailboats, such as those used in the America’s Cup races, have deep, narrow keels that are very efficient at providing the lift necessary to prevent sidslipping. Of course, such keels are a problem for recreational sailors in shallow waters.

### Resistive forces

A sailboat experiences several kinds of resistance. The first is simply the resistance of the hull moving through water. As the boat moves, it shears the water. Water molecules adhere to the hull’s surface. So there must be a shear—that is, a velocity gradient—between the adhering molecular layer at rest

with respect to the hull and the bulk of water farther away. The shear means that van der Waals couplings between water molecules are being broken. That costs energy and creates the resistive force, which becomes stronger as the boat’s speed increases. The energy dissipation also increases with the total area of wetted surface.

Although the effect is called frictional resistance, it’s important to realize that the resistive force in water is basically different from the frictional force between solid surfaces rubbed together. To reduce ordinary friction, one can polish or lubricate the sliding surfaces. That makes surface bumps smaller, and it substitutes the shearing of fluid lubricant molecules for shearing of the more tightly bound molecules on the solid surfaces.

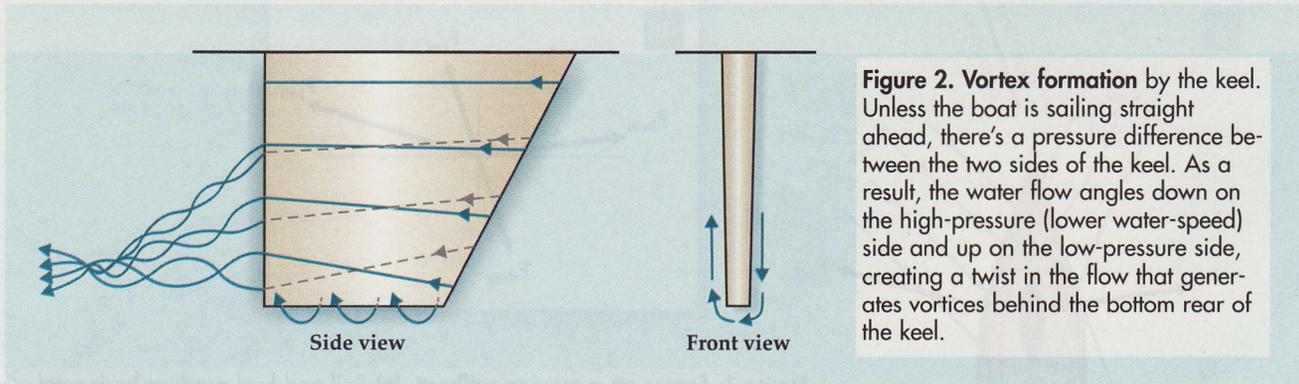
For a boat moving through water, however, polishing the hull doesn’t eliminate the shearing of the molecules of water, which is already a fluid. The resistive force cannot be reduced significantly except by reducing the wetted surface. It does help to have a smooth surface, but that’s primarily to reduce turbulence.

The generation of turbulence is a general phenomenon in the flow of fluids. At sufficiently low speeds, fluid flow is laminar. At higher speeds, turbulence begins. Its onset has to do with the shearing of the molecules in the fluid. When the shearing reaches a critical rate, the fluid can no longer respond with a continuous dynamic equilibrium in the flow, and the result is turbulence. Its onset is quantified in terms of the Reynolds number

$$R = (Lv)/(\mu/\rho), \quad (1)$$

where  $v$  is the velocity of the flowing fluid,  $\mu$  is its viscosity,  $\rho$  is its density, and  $L$  is the relevant length scale of the system. Rearranging factors in equation 1, one can think of  $R$  as the ratio of inertial forces ( $\rho v$ ) to viscous forces ( $\mu/L$ ). In the late 19th century, English engineer Osborne Reynolds found that, with surprising universality, turbulence begins when that dimensionless parameter exceeds about a million.

For a boat of length  $L$  moving through water at velocity  $v$  to see when turbulence begins in the flow along the hull,  $R$  is about  $10^6 Lv$  (in SI units). A typical speed for a sailboat is 5 knots (2.4 m/s). At that speed, then, one should expect turbulence for any boat longer than half a meter. (Used worldwide



**Figure 2. Vortex formation** by the keel. Unless the boat is sailing straight ahead, there's a pressure difference between the two sides of the keel. As a result, the water flow angles down on the high-pressure (lower water-speed) side and up on the low-pressure side, creating a twist in the flow that generates vortices behind the bottom rear of the keel.

as a measure of boat speed, a knot is one nautical mile per hour. A nautical mile is one arcminute of latitude, or 1.85 km.)

Because turbulence dissipates energy, it increases the resistance to motion through the water. With turbulence, a sailboat's resistance is typically four or five times greater than it is when the flow along the hull is laminar. A rough surface will cause turbulence to be greater and begin sooner. That's the main reason to have a smooth hull surface.

Turbulence also occurs in the air flowing along the surface of the sail. Water is a thousand times denser than air and 50 times more viscous. So for the air-sail system one gets

$$R = 7 \times 10^4 Lv. \quad (2)$$

For a typical wind speed of 5 m/s, then, one gets turbulence if the sail is wider than about 3 meters. When turbulence forms in the air flow along the sail, the desired pressure difference between the two sides of the sail—its lift—is diminished.

Another important resistive force comes from vortex generation at the bottom of the keel and at the top of the sails. When the air or water moves around the longer-path side of the sail or keel, its speed increases and therefore its pressure falls. As the air or water moves along the sail or keel, it will respond to the resulting pressure difference by trying to mi-

grate from the high-pressure side to the low-pressure side. Figure 2 sketches that effect for a keel. What actually happens, as shown in the figure's side view, is that the flow angles a bit up on one side and down on the other. When those flows meet at the back of the sail or keel, the difference in their arrival angles has a twisting effect on the fluid flow that can cause a vortex to come off the top of the sail or the bottom of the keel.

The effect is well known for airplane wings. Called induced drag, vortex formation costs energy. Figure 3 shows vortices generated at the tops of sails by racing sailboats moving through a fog. A long keel will generate very large vortices. By making the keel short and deep, one can increase the ratio of lift to energy dissipated by vortices. The same is accomplished—especially for sailboats racing upwind—by having tall, narrow sails. It's also why gliders have long, narrow wings.

Because it's often impractical to have a short, deep keel or a narrow, long wing, one can install a vane at the tip to reduce the flow from the high-pressure to the low-pressure side. On planes they're called winglets, and on keels they're simply called wings. A modern recreational or cruising sailboat will have a keel that's a compromise between the old-



**Figure 3. Sailtops** form vortices visible in fog. The boats were participating in the 2001–02 Volvo Ocean Race off Cape Town, South Africa. (Photo copyright Daniel Forster, used with permission.)



**Figure 4.** Moving at hull speed, a sailboat generates a bow wave whose wavelength just equals the length of the boat's water line. The wave crests at bow and stern, with a single well-formed trough in between. (Photo by Greg Green, used with permission.)

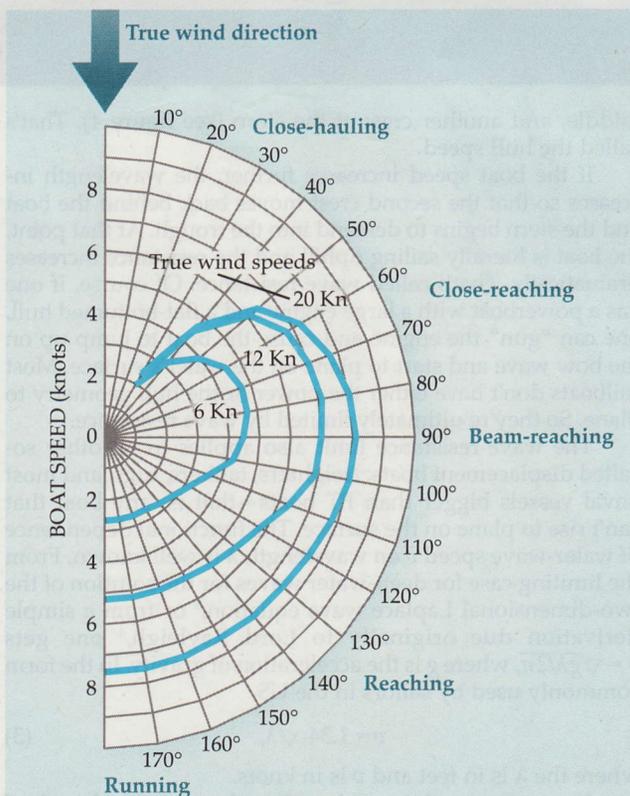
fashioned long keels and the modern deep, narrow keels—with a wing at the bottom rear end to reduce induced drag. Such keel wings were first used by the victorious sailboat *Australia II* in the 1983 America's Cup race. Modern wing theory also suggests that to minimize induced drag, keels and sails should have elliptic or tapered trailing edges.<sup>2</sup> Such shaped edges are now common.

A sailboat also has a resistance component due simply to its deflection of water sideways as it advances. That's called form resistance, and it obviously depends on hull geometry. It's easy to see that narrow hulls provide less resistance than do wider hulls. Any boat will always be a compromise between providing low form resistance and providing passenger and cargo space. Seeking to minimize form resistance for a given hull volume, shipbuilders have tried many basic hull shapes over the centuries. Even Isaac Newton weighed in on the question. He concluded that the best hull shape is an ellipsoid of revolution with a truncated cone at the bow.

Extensive computer modeling and tank testing have resulted in a modern hull design that widens slowly back from the bow and then remains fairly wide near the stern. Even with a wide stern, designers try to provide enough taper toward the back to allow smooth flow there. That taper is often accomplished by having the stern rise smoothly from the water rather than by narrowing the beam. If the flow from the stern is not smooth, large eddies will form and contribute to resistance.

### Hull speed

As a boat moves through water, it creates a bow wave that moves with the speed of the boat. Water waves are dispersive; long waves propagate faster than short ones. Therefore the length of the full wave generated by the bow is determined by the boat's speed. As a boat starts to move slowly through the water, one sees at first a number of wave crests and troughs moving down the side of the hull. As the boat speeds up, the wavelength gets longer and one sees fewer waves down the side. Eventually at some speed, the wave will be long enough so that there's just one wave down the side of the boat, with its crest at the bow, a trough in the



**Figure 5.** Speeds predicted by a computer model<sup>5</sup> for a 10-meter-long cruising sailboat, plotted for three different wind speeds from 6 to 20 knots as a function of the angle of the boat's motion relative to the wind direction. (10 knots = 18.5 km/h.) An angle of 180° means the boat is "running" with the wind directly at its back. The fastest speeds are predicted when the boat is "beam reaching," that is, moving at about 90° to the wind. The boat even makes some progress when it's "close hauling" almost directly into the wind.



**Figure 6.** A hydrofoil sailboat with solid, winglike sails, moving at about twice the wind speed with the wind abeam—that is, blowing from the side. (Copyright École Nationale Supérieure de Techniques Avancées.)

middle, and another crest at the stern (see figure 4). That's called the hull speed.

If the boat speed increases further, the wavelength increases so that the second crest moves back behind the boat and the stern begins to descend into the trough. At that point, the boat is literally sailing uphill and the resistance increases dramatically. That's called wave resistance. Of course, if one has a powerboat with a large engine and a flat-bottomed hull, one can "gun" the engine and cause the boat to jump up on the bow wave and start to plane on the water's surface. Most sailboats don't have either the power or the hull geometry to plane. So they're ultimately limited by wave resistance.

The wave-resistance limit also applies to all other so-called displacement boats: freighters, tankers, tugs, and most naval vessels bigger than PT boats—that is, any boat that can't rise to plane on the surface. The functional dependence of water-wave speed  $v$  on wavelength  $\lambda$  is well known. From the limiting case for deep-water waves for the solution of the two-dimensional Laplace wave equation,<sup>3</sup> or from a simple derivation due originally to Lord Rayleigh,<sup>4</sup> one gets,  $v = \sqrt{g\lambda/2\pi}$ , where  $g$  is the acceleration of gravity. In the form commonly used by sailors in the US,

$$v = 1.34 \sqrt{\lambda}, \quad (3)$$

where the  $\lambda$  is in feet and  $v$  is in knots.

If one equates the wavelength to the waterline length of a boat, equation 3 gives the boat's hull speed. For a sailboat with a waterline length of 20 feet (6 m), the hull speed is 6 knots. For a large cruising sailboat with a waterline of 40 feet (12 m), it's about 8 knots. And for a 300-foot-long naval vessel, it's 23 knots. In practice, it's very difficult to make a displacement boat go faster than about 1.5 times its hull speed.

Combining all the components of resistance for a sailboat moving at close to its hull speed, one finds that the frictional resistance contributes about a third of the total, and the wave resistance another third. Form resistance accounts for about 10%, as does the induced drag from vortex generation

at the bottom of the keel. The assorted remaining contributions, including eddy formation behind the boat and aerial vortex generation by the sails, provide the remaining 10 to 15%. Of course the fractional contributions vary with boat speed, wave conditions, and the direction of motion relative to the wind.

### Predicting speed

One can exploit the physics of sailing to calculate boat speeds for a given sailboat for different wind speeds and points of sail. Such calculations are usually performed iteratively by computer programs that start from two basic vector equations to be solved simultaneously:

$$\mathbf{F}_{\text{drive}} = \mathbf{F}_{\text{resistance}} \quad \text{and} \quad \mathbf{M}_{\text{heel}} = \mathbf{M}_{\text{righting}}. \quad (4)$$

Here  $\mathbf{F}_{\text{drive}}$  is the total driving force in the direction of motion provided by the wind in the sails, and  $\mathbf{F}_{\text{resistance}}$  is the sum of all the resistive forces. The torques  $\mathbf{M}_{\text{heel}}$  and  $\mathbf{M}_{\text{righting}}$  are the heeling and righting moments caused by the wind in the sails and the weight of the hull and keel.

The force of the wind on the sail is calculated as a lifting force perpendicular to the apparent wind direction and a drag force in the direction of the apparent wind. (The apparent wind is the wind as perceived by an observer aboard the moving vessel.) These lift and drag forces are then resolved into components along and perpendicular to the direction of motion. The net force in the direction of motion is then  $\mathbf{F}_{\text{drive}}$  and the net force perpendicular to the boat's motion is what produces the heeling moment. The two equations in (4) must be solved simultaneously because the angle of heel affects the total driving force.

Following Bernoulli's principle, one takes the force of the wind in the sails to be proportional to the total sail area times the square of the apparent wind speed. The actual forces are then obtained with empirical lift and drag coefficients, given as functions of sail geometry and angle of attack. Frictional resistance is proportional to the hull's wetted surface area and

increases as the square of the boat's speed. All the various contributions to total resistance involve empirical coefficients. Wave and form resistance are expressed as functions of the hull's "prismatic coefficient," which is an inverse measure of the tapered slimness of its ends.

There are simple and complex speed-prediction computer programs. Some that have been refined over decades for racing applications are kept private and closely guarded. Figure 5 shows the results of calculations I performed for a 30-foot (10-m) cruising sailboat using a publicly available program.<sup>5</sup> The figure shows the calculated boat speed as a function of wind speed and point of sail. The predicted boat speeds are greatest when one is sailing about 90° away from the wind direction. Sailors call that beam reaching. It yields a boat speed of about half the wind speed.

Such calculations are confirmed experimentally, with a degree of accuracy that depends on the sophistication of the model and on how much the program has been tuned for a specific kind of sailboat. Broadly speaking, a sailboat is faster if it is longer and narrower, with bigger sails and a smaller wetted surface. Such general rules can, of course, yield a boat that's longer than one wants, or tips over too easily, or has too little room inside.

So every design feature is a compromise between competing needs. For sailing downwind, one wants fairly square sails, which are best at catching the wind. But for sailing upwind, taller, narrower sails are best, because they maximize the ratio of lift to energy lost by generating vortices. The most efficient keel is deep and narrow, to maximize lift with minimal surface area. But a deep keel is problematic in shallow waters. Shorter keels with wings or bulbs at the bottom usu-

ally represent the best compromise for overall sailing.

What's the highest speed a sailboat can reach? The trick is to reduce resistance. An iceboat can outrun the wind because it has so little resistance. For a sailboat, the resistance comes primarily from having to plow through the water. The best way to reduce that resistance is to move less and less of the boat through the water. One answer is hydrofoils. They are vanes placed below the hull that raise it out of the water as the boat speeds up.

Sailboats with hydrofoils have reached speeds of more than 40 knots when the wind speed was barely half that. One such craft is shown in figure 6. These vessels are not usually practical for cruising and other normal recreational activities. They're sometimes dismissed as low-flying aircraft. A more practical alternative is the catamaran—a double-hulled sailboat. Catamarans are being developed to provide relatively stable, fast sailing. Although they are more expensive than traditional single-hull sailboats for a given amount of living space, catamarans are becoming increasingly popular.

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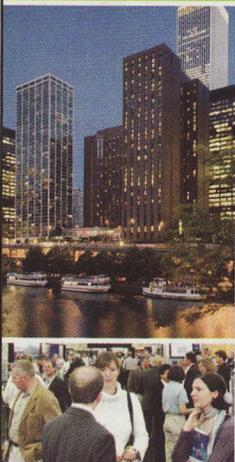
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