Orbital Coordinate System Conversion

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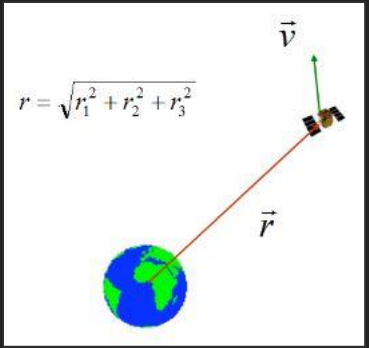


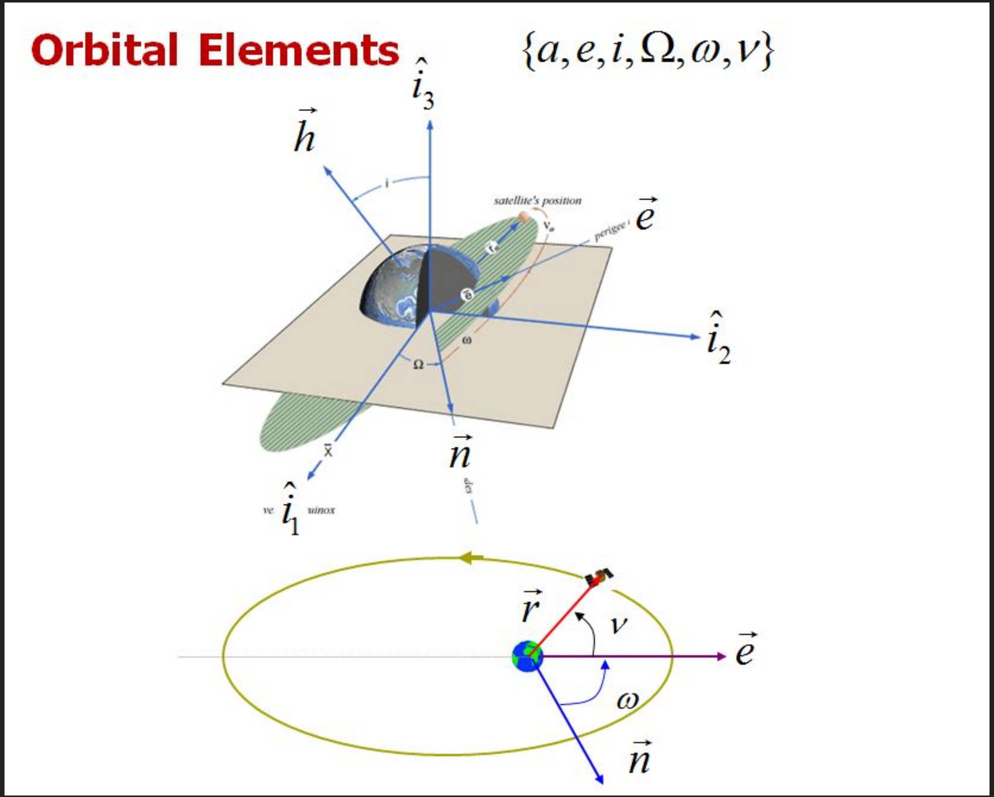
Orbital Background

$$ {}^{i}\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = 0 $$

$$ \vec{r} = r_1\hat{i}_1 + r_2\hat{i}_2 + r_3\hat{i}_3 $$

$$ \vec{v} = \dot{r}_1\hat{i}_1 + \dot{r}_2\hat{i}_2 + \dot{r}_3\hat{i}_3 $$
$$ = v_1\hat{i}_1 + v_2\hat{i}_2 + v_3\hat{i}_3 $$

$$ r^{i} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \qquad v^{i} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} $$



Coordinate Conversion

- Six orbital parameters
- Orbital to Cartesian coordinate conversion (Algorithm 6.0)
- Use output as input for opposite conversion (Algorithm 5.0), with only <u>0.0001</u> error.

ALGORITHM 6: RANDV $(p, e, i, \Omega, \omega, \nu\ (u, \lambda_{true}, \tilde{\omega}_{true}) \Rightarrow \vec{r}_{IJK}\ \vec{v}_{IJK})$

IF Circular Equatorial

SET $(\omega, \Omega) = 0.0$ and $\nu = \lambda_{true}$

IF Circular Inclined

SET $\omega = 0.0$ and $\nu = u$

IF Elliptical Equatorial

```
function [r,v]=orb2rv(p,e,i,W,w,nu)

% [r,v]=rv2orb(p,e,i,W,w,nu) %...%

mu=1; %mu=GM

nu=nu*pi/180;
snu=sin(nu); cnu=cos(nu);
mup=sqrt(mu/p);

s_r=p/(1+e*cnu);
r_o=[s_r*cnu s_r*snu 0]'; % in orbital frame
v_o=[-(mup*snu) (mup*(e+cnu)) 0]';

C=DCMeci2orb(i,W,w);
r=C*r_o;
v=C*v_o;
```

Orbital Data

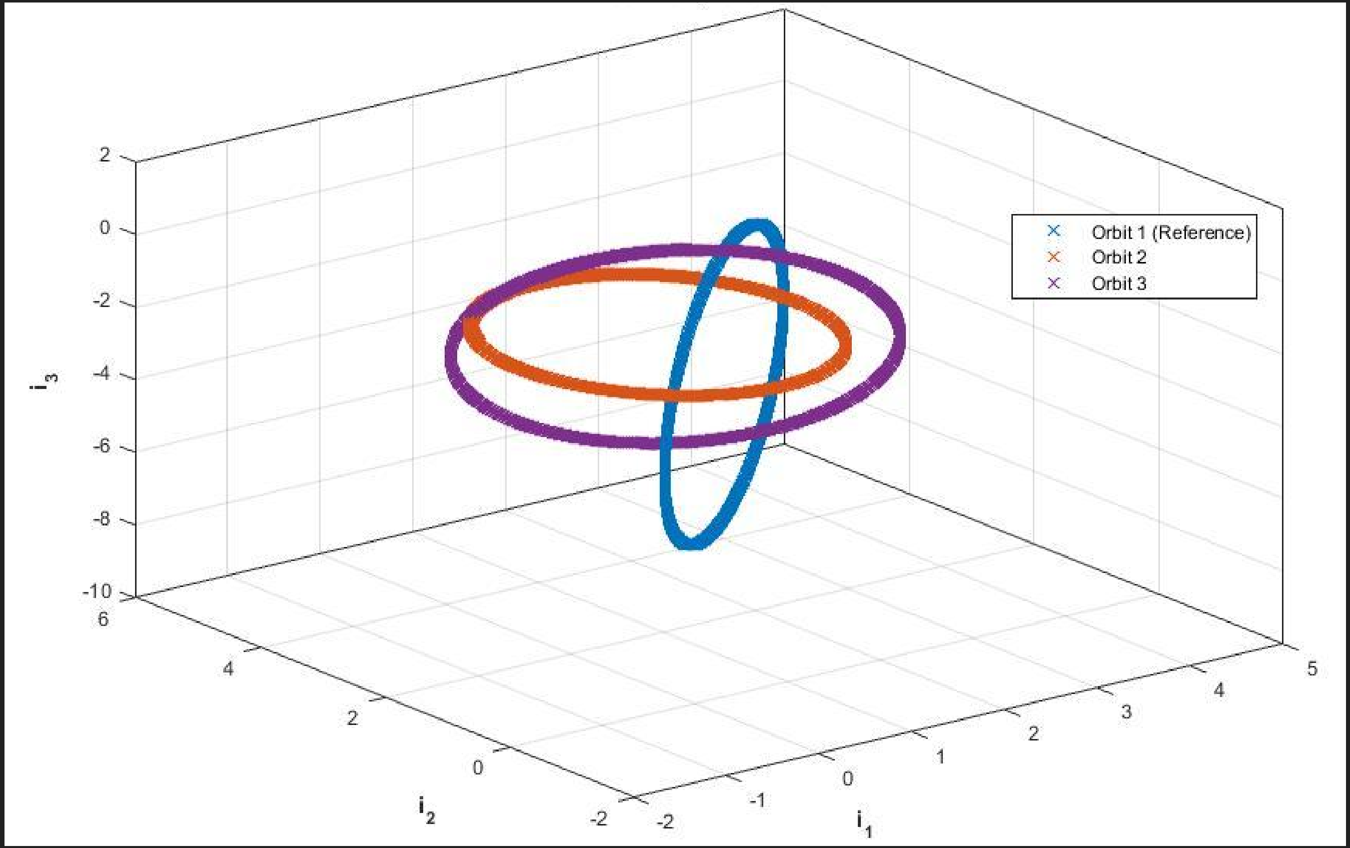
1. Reference:
   r1=[1.023; 1.076; 1.011]; v1=[0.62; 0.70; -0.25]
2. User #1:
   r2=[1; 1.5; 0.1]; v2=[-0.5; 0.5; 0.2]
3. User #2:
   r3=[0.4762; 0.8248; 1.6496];
   v3=[-0.6277; 0.3977; 0.0530];

Orbital Plots

1. Reference

2. User #1

3. User #2



Mahalo :)