2017 HESTEMP Conference: Instantaneous Screw Axis (ISA)

Michael Poon

Advisor: Dr. Dilmurat Azimov
Consultant: Ted Ralston

Contact Info:
azimov@hawaii.edu
evankawa@hawaii.edu

College of Engineering
University of Hawai'i at Mānoa

Introduction

Any rigid body can be described by an instantaneous screw axis

The instantaneous translational and rotational motion can be found about the instantaneous screw axis.

Two of the invariants correspond to rotational velocity about and translational velocity along the ISA

The other four invariants correspond to rotational and translational velocities, which model the spatial motion of the ISA.

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De Schutter, J. (2010, February). Invariant Description of Rigid Body Motion Trajectories. *Journal of Mechanisms and Robotics*, *2*, 1-8

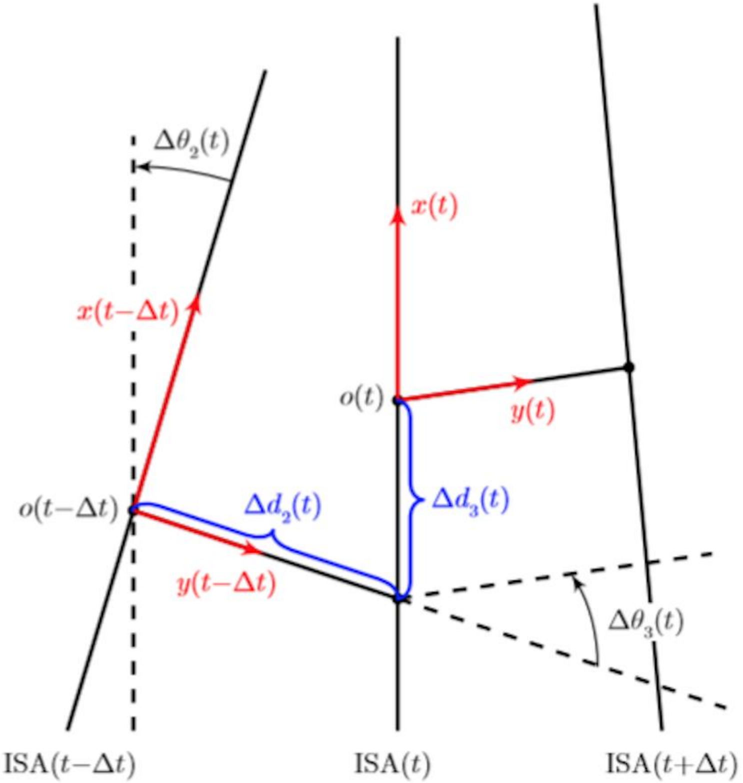

Fig. 1: Instantaneous screw axes at three different moments in time

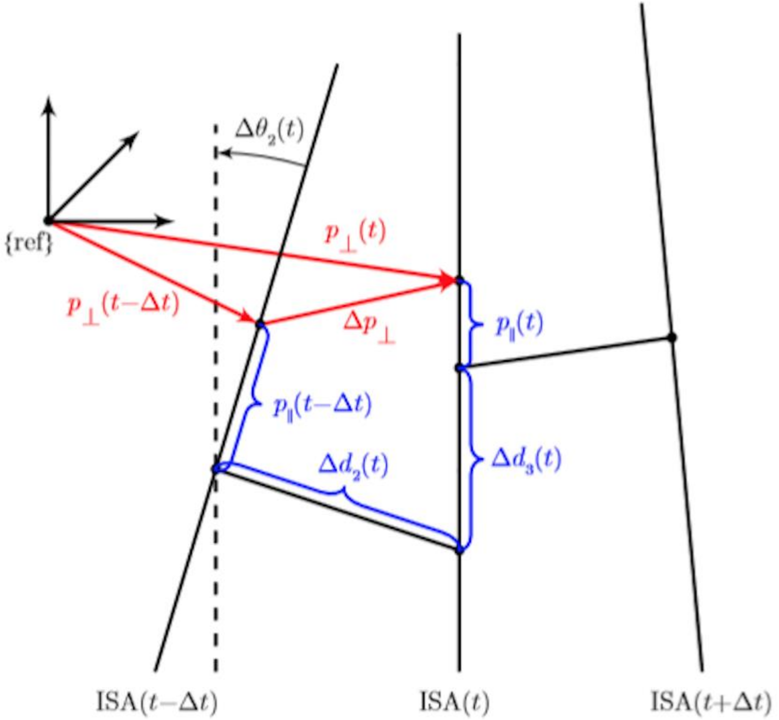

Fig. 2: The position vector from the origin of {ref} and perpendicular to the ISA.

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Calculations

$$e_x = \pm \frac{\boldsymbol{\omega}}{\|\boldsymbol{\omega}\|} \qquad e_y = \pm \frac{\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}}{\|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|} \qquad e_z = \pm \frac{\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}})}{\|\boldsymbol{\omega}\| \cdot \|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|}$$

$$\omega_1 = \pm\|\boldsymbol{\omega}\| \cdot \boldsymbol{e}_x$$

$$\omega_2 = \pm \frac{\|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|}{\|\boldsymbol{\omega}\|^2} \cdot \boldsymbol{e}_y$$

$$\omega_3 = \pm \frac{(\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}) \times (\boldsymbol{\omega} \times \ddot{\boldsymbol{\omega}})}{\|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|^2} \cdot \boldsymbol{e}_x$$

$$v_1 = \boldsymbol{v} \cdot \boldsymbol{e}_x$$

$$v_2 = \pm \frac{(\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}})}{\|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|} \cdot \left[\frac{(\dot{\boldsymbol{\omega}} \times \boldsymbol{v} + \boldsymbol{\omega} \times \dot{\boldsymbol{v}}) \cdot \|\boldsymbol{\omega}\|^2 - 2(\boldsymbol{\omega} \times \boldsymbol{v}) \cdot (\boldsymbol{\omega} \cdot \dot{\boldsymbol{\omega}})}{\|\boldsymbol{\omega}\|^4}\right]$$

$$v_3 = \boldsymbol{e}_x \cdot \dot{\boldsymbol{p}}_\perp - \boldsymbol{p}_\parallel$$

$$p_\parallel = -\frac{\boldsymbol{e}_z \cdot \dot{\boldsymbol{p}}_\perp}{\omega_2} \qquad p_\perp = \frac{\boldsymbol{\omega} \times \boldsymbol{v}}{\|\boldsymbol{\omega}\|^2}$$

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Calculations

Closed-Form Expression for $\boldsymbol{v}_3$

$$\boldsymbol{v}_3 = \mp \frac{[\dot{\boldsymbol{\omega}} \times (\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \ddot{\boldsymbol{\omega}})] \cdot \{\|\boldsymbol{\omega}\|^2 \cdot (\dot{\boldsymbol{\omega}} \times \boldsymbol{v} + \boldsymbol{\omega} \times \dot{\boldsymbol{v}}) - 2\boldsymbol{\omega} \cdot \dot{\boldsymbol{\omega}} \cdot (\boldsymbol{\omega} \times \boldsymbol{v})\}}{\|\boldsymbol{\omega}\|^3 \cdot \|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|^2}$$

$$\mp \frac{(\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}})) \cdot \{\|\boldsymbol{\omega}\|^2 \cdot (\ddot{\boldsymbol{\omega}} \times \boldsymbol{v} + 2\dot{\boldsymbol{\omega}} \times \dot{\boldsymbol{v}} + \boldsymbol{\omega} \times \ddot{\boldsymbol{v}}) - 2\ (\|\dot{\boldsymbol{\omega}}\|^2 + \boldsymbol{\omega} \cdot \dot{\boldsymbol{\omega}})(\boldsymbol{\omega} \times \boldsymbol{v}))\}}{\|\boldsymbol{\omega}\|^3 \cdot \|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|^2}$$

$$\pm \left[\frac{3}{2} \cdot \frac{\boldsymbol{\omega} \cdot \dot{\boldsymbol{\omega}}}{\|\boldsymbol{\omega}\|^2} + \frac{(\boldsymbol{\omega}\ .\ \times \dot{\boldsymbol{\omega}}) \cdot (\boldsymbol{\omega} \times \ddot{\boldsymbol{\omega}})}{\|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|^2}\right] \cdot \frac{(\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}})) \cdot \{\|\boldsymbol{\omega}\|^2 \cdot (\dot{\boldsymbol{\omega}} \times \boldsymbol{v} - \boldsymbol{\omega} \times \dot{\boldsymbol{v}}) - 2(\boldsymbol{\omega} \cdot \dot{\boldsymbol{\omega}})(\boldsymbol{\omega} \times \boldsymbol{v})\}}{\|\boldsymbol{\omega}\|^3 \cdot \|\boldsymbol{\omega} \times \dot{\boldsymbol{\omega}}\|^2}$$

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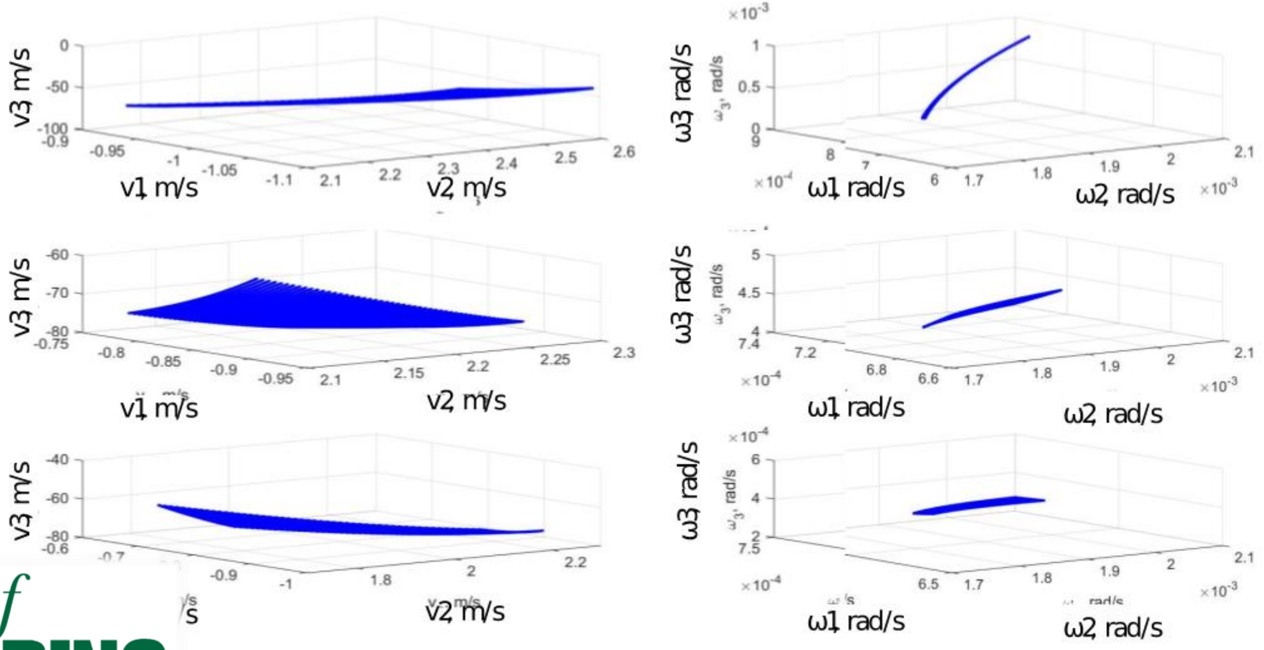
3-D Plots of ISA Invariants Envelope



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Conclusion

On-going research project

To be used with autonomous flight controller (i.e. Pixhawk) in order for full guidance and control of a UAV

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