Financial Engineering of the Integration of Global Supply Chain Networks and Social Networks with Risk Management

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Abstract: In this paper, we focus on the financial engineering of integrated global supply chain networks and social networks. Through a multilevel, dynamic supernetwork framework consisting of the global supply chain network with electronic commerce and the social network, we capture the multicriteria decision-making behavior of the various decision-makers (manufacturers, retailers, and consumers), which includes the maximization of profit, the maximization of relationship values, and the minimization of risk. Relationship levels in our framework are assumed to influence transaction costs as well as risk and to have value for the decision-makers. We explore the dynamic co-evolution of the global product transactions, the associated product prices, and the relationship levels on the supernetwork until an equilibrium pattern is achieved. The pricing mechanism guarantees that optimality for each decision-maker is achieved in the financially engineered multitiered, multilevel supernetwork. We provide some qualitative properties of the dynamic trajectories, under suitable assumptions, and propose a discrete-time algorithm, which yields explicit closed form expressions at each iteration for the tracking of the evolution of the global product transactions, relationship levels, and prices until an equilibrium is attained. We illustrate the model and computational procedure with several numerical examples.

1. Introduction

The landscape for global supply chain decision-making has been transformed through advances in telecommunications and, in particular, the Internet, with the creation of new linkages among suppliers, manufacturers, retailers, and consumers both within a country as well as beyond national borders. Indeed, the advent of electronic commerce is enabling the world to move closer to the realization of a single, borderless market and is driving the increasing globalization of not only businesses but also supply chains.

The importance of electronic commerce (e-commerce) in global supply chains has been recognized not only in practice but also increasingly by researchers, including, among others, Kogut and Kulatilaka [28], Vidal and Goetschalckx [45], and Cohen and Huchzermeier [8]. Nagurney, Cruz, and Matsypura [30] developed a global supply chain network model with electronic commerce which allowed for the dynamic tracking of the product trajectories and prices over time. Additional references to supply chains and e-commerce may be found in the books by Bramel and Simchi-Levi [5] and by Nagurney and Dong [31], and in the edited volume by Simchi-Levi, Wu, and Shen [43].

At the same time that globalization and e-commerce have created new opportunities for economic transactions, global supply chains are increasingly exposed to new risks and uncertainties ranging from the threats of illnesses such as SARS (cf. [14]), which seriously disrupted supply chains, to terrorist threats and wars (see, e.g., [41]). Frameworks for risk management in a global supply chain context, but with a focus on centralized decision-making and optimization, have been proposed in [8], [9], [21], and the references therein.

In this paper, we focus on the financial engineering of global supply chains by introducing explicitly social networks within a supernetwork perspective. Financial engineering, as defined by the International Association of Financial Engineering (IAFE) [23], is the application of various mathematical, statistical, and computational techniques to solve practical problems in finance. According to IAFE, such problems include the valuation of financial instruments such as options, futures, and swaps; the trading of securities; risk management, and the regulation of financial markets. For background on financial engineering, we refer the reader to Birge and Linetsky [3, 4]. Guenes and Pardalos [19] present an annotated bibliography of network optimization in supply chains and financial engineering. The book edited by Pardalos and Tsitsiringos [39] contains additional references linking financial engineering, supply chains, and e-commerce.

Specifically, in this paper, we apply the concept of financial engineering to the process of supply chain risk management through the inclusion of relationship levels. Kelly [27] argued that "the network economy is founded on technology, but it can only be built on relationships." Moreover, Castells [7] noted that technology and globalization are making networks of relationships a critical business asset. Spekman and Davis [44] found that supply chain networks that exhibit collaborative behaviors tend to be more responsive and that supply chain-wide costs are, hence, reduced. These results are also supported by Dyer [13] who demonstrated empirically that a higher level of trust (relationship) lowers transaction costs (costs associated with negotiating, monitoring, and enforcing contracts). Baker and Faulkner [1] present an overview of papers by economic sociologists that show the important role of relationships due to their potential to reduce risk and uncertainty.

Wakolbinger and Nagurney [46] recently developed a dynamic supernetwork framework for the modeling and analysis of supply chains with electronic commerce that included the role that relationships play. That contribution was apparently the first to introduce relationship levels in terms of flows on networks, along with logistical flows in terms of product transactions, combined with pricing. The concept of relationship levels was inspired by a paper by Golicic, Foggin, and Mentzer [17] who introduced the concept of relationship magnitude. That research strongly suggested that different relationship magnitudes lead to different benefits and that different levels of relationship magnitudes can be achieved by putting more or less time and effort into the relationship. The idea of a continuum of relationship strength is also supported by several theories of relationship marketing that suggest that business relationships vary on a continuum from transactional to highly relational (cf. [15]). The model by Wakolbinger and Nagurney [46] operationalized the frequently mentioned need to create a portfolio of relationships (cf. [6], [17]). The "optimal" portfolio balanced out the various costs and the risk, against the profit and the relationship value and included the individual decision-maker's preferences and risk aversions.

The model of Wakolbinger and Nagurney [46], however, dealt with a single country and a single currency. The extension to an international setting is timely since global supply chain transactions can be expected to be potentially riskier than single country ones due to cultural differences, difficulties with languages, and distances, and, hence, higher relationship levels may be of even greater significance and may create competitive advantages. Hogan [20], among others, has argued for the need to develop quantifiable measures of relationship value in the globally competitive marketplace as well as a theoretical framework of business-to-business relationships.

This paper models, in a global context, the multicriteria decision-making behavior of the various decision-makers in a supply chain network, which includes the maximization of profit, the minimization of risk, and the maximization of relationship values through the inclusion of the social network, in the presence of both business-to-business (B2B) and business-to-consumer (B2C) transactions. Moreover, we explicitly describe the role of relationships in influencing transaction costs and risk. In addition, we subsume other (potenital) benefits of relationships under the term *relationship value* ([20], [35]) and we introduce general relationship value functions. Both the risk functions and the relationship value functions are allowed to depend, in general, upon the quantities of the product transacted between the various decision-makers as well as on the relationship levels. Hence, we truly capture the networks of relationships in the global supply chain framework.

This paper is organized as follows. In Section 2, we develop the multilevel supernetwork model consisting of multiple tiers of decision-makers acting on the global supply chain network with electronic transactions and on the social network. We describe the decision-makers' optimizing behavior, and establish the governing equilibrium conditions along with the corresponding variational inequality formulation. In Section 3, we propose the disequilibrium dynamics of the global product transactions, the prices, and the relationship levels as they co-evolve over time given the initial conditions. We then formulate the dynamics as a projected dynamical system and establish that the set of stationary points of the projected dynamical system coincides with the set of solutions to the derived variational inequality. In Section 4, we present a discrete-time algorithm which approximates (and tracks), under the appropriate assumptions, the evolution of the global product transactions, prices, and the relationship level trajectories over time until the equilibrium values are reached. We then apply the discrete-time algorithm in Section 5 to several numerical examples to further illustrate the model and computational procedure. We conclude with Section 6, in which we summarize our results and present our conclusions.

2. The Supernetwork Model Integrating Global Supply Chain Networks with Electronic Commerce and Social Networks

In this Section, we develop the supernetwork model with manufacturers, retailers, and demand markets in a global context in which we explicitly integrate social networks and also include electronic commerce. We focus here on the presentation of the model within an equilibrium context, whereas in Section 3, we provide the disequilibrium dynamics and the co-evolution of the global supply chain product transactions, the prices, as well as the relationship levels between tiers of decision-makers over time. This model significantly generalizes the model of Nagurney, Cruz, and Matsypura [30] to explicitly include social networks as well as electronic transactions between manufacturers and demand markets and between retailers and demand markets. In addition, the supernetwork model in its static and dynamic forms (cf. Section 3) broadens the framework proposed in Wakolbinger and Nagurney [46] to the global dimension and introduces more general risk and relationship value functions.

We assume that the manufacturers are involved in the production of a homogeneous product and we consider L countries, with I manufacturers in each country, and J retailers, which are not country-specific but, rather, can be either physical or virtual, as in the case of electronic commerce. There are K demand markets for the homogeneous product in each country and H currencies in the global economy. We denote a typical country by l or $\hat{l}$, a typical manufacturer by i, and a typical retailer by j. A typical demand market, on the other hand, is denoted by k and a typical currency by h. We assume that each manufacturer can transact directly electronically with the consumers at the demand markets through the Internet and, for the sake of generality, he can also conduct transactions with the retailers either physically or electronically in different currencies. The demand for the product in a country can be associated with a particular currency. We let m refer to a mode of transaction with $m = 1$ denoting a physical transaction and $m = 2$ denoting an electronic transaction via the Internet.

The depiction of the supernetwork is given in Figure 1. As this figure illustrates, the supernetwork is comprised of the social network, which is the bottom level network, and the global supply chain network, which is the top level network. Internet links to denote the possibility of electronic transactions are denoted in the figure by dotted arcs. In addition, dotted arcs/links are used to depict the integration of the two networks into a supernetwork. Examples of other supernetworks

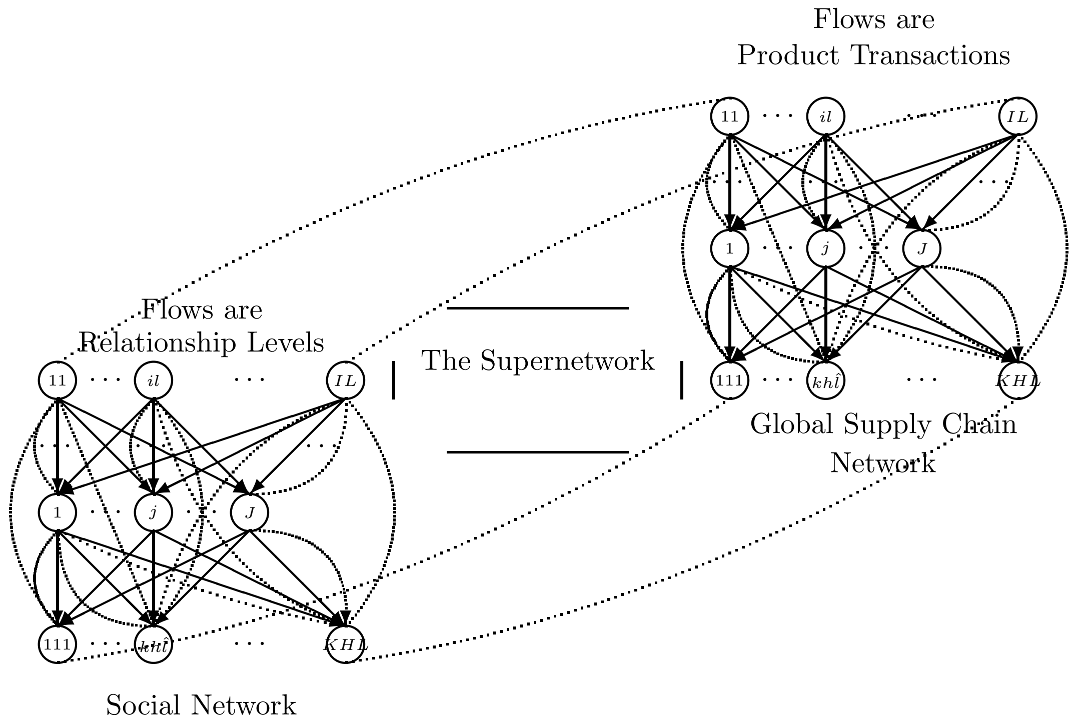

Figure 1: The Multilevel Supernetwork Structure of the Integrated Global Supply Chain Network / Social Network System

can be found in Nagurney and Dong [31]. Subsequently, we describe the interrelationships between the global supply chain network and the social network through the functional forms and the flows on the links.

The supernetwork in Figure 1 consists of a social network and a global supply chain network with each network consisting of three tiers of decision-makers. The top tier of nodes in each network consists of the manufacturers in the different countries, with manufacturer i in country l being referred to as manufacturer il and associated with node il. There are, hence, IL top-tiered nodes in each network. The middle tier of nodes in each of the two networks consists of the retailers (which recall need not be country-specific) and who act as intermediaries between the manufacturers and the demand markets, with a typical retailer j associated with node j in this (second) tier of nodes. The bottom tier of nodes in both the social network and in the supply chain network consists of the demand markets, with a typical demand market k in currency h and country $\hat{l}$, being associated with node $kh\hat{l}$ in the bottom tier of nodes. There are, as depicted in Figure 1, J middle (or second) tiered nodes corresponding to the retailers and KHL bottom (or third) tiered nodes in the global supply chain network and in the social network.

We have identified the nodes in the supernetwork and now we turn to the identification of the links joining the nodes in a given tier with those in the next tier. We first focus on the global supply chain network. We assume that each manufacturer i in country l involved in the production of the homogeneous product can transact with a given retailer in any of the H available currencies, as represented by the H links joining each top tier node with each middle tier node j; $j = 1, \ldots, J$. A manufacturer may also transact with consumers at a demand market directly via the Internet. Furthermore, each retailer (intermediate) node j; $j = 1, \ldots, J$, can transact with each demand market denoted by node $kh\hat{l}$. The product transactions represent the flows on the links of the supply chain network in Figure 1.

We construct analogous links on the social network component of the supernetwork. We assume that each manufacturer i in country l can establish a certain relationship level with a given retailer in any of the H available currencies, as represented by the H links joining each top tier node with each middle tier node j; $j = 1, \ldots, J$. A manufacturer may also establish a relationship level with consumers at a demand market directly via the Internet. Furthermore, each retailer (intermediate) node j; $j = 1, \ldots, J$, can establish a relationship level with a demand market denoted by node $kh\hat{l}$. We assume that the relationship levels are nonnegative and that they may attain a value from 0 through 1. A relationship level of 0 indicates no relationship between the two decision-makers, a relationship of 1 indicates the highest possible relationship. These relationship levels represent the flows on the social network in Figure 1.

Note that there will be prices associated with each of the tiers of nodes in the global supply chain network. The model also includes the rate of appreciation of currency h against the basic currency, which is denoted by e_h (see [30]). These "exchange" rates are grouped into the column vector $e \in R^H$. The variables for this model are given in Table 1. All vectors are assumed to be column vectors.

Table 1: Variables in the Integrated Global Supply Chain Network / Social Network System

Notation	**Definition**
q	IL-dimensional vector of the amounts of the product produced by the manufacturers in the countries with component il denoted by q^{il}
Q^1	$2ILJH$-dimensional vector of the amounts of the product transacted between the manufacturers in the countries in the currencies with the retailers via the two modes with component ${}^{il}_{jhm}$ denoted by q^{il}_{jhm}
Q^2	$ILKHL$-dimensional vector of the amounts of the product transacted between the manufacturers in the countries in the currencies and the demand markets with component ${}^{il}_{kh\hat{l}}$ denoted by $q^{il}_{kh\hat{l}}$
Q^3	$2JKHL$-dimensional vector of the amounts of the product transacted between the retailers and the demand markets in the countries and currencies via the two modes with component ${}^{j}_{kh\hat{l}m}$ denoted by $q^{j}_{kh\hat{l}m}$
η^1	$2ILJH$-dimensional vector of the relationships levels between the manufacturers in the countries and the retailer/currency/mode combinations with component ${}^{il}_{jhm}$ denoted by η^{il}_{jhm}
η^2	$ILKHL$-dimensional vector of the relationships levels between the manufacturers in the countries and the demand market/ country/currency combinations with component ${}^{il}_{kh\hat{l}}$ denoted by $\eta^{il}_{kh\hat{l}}$
η^3	$2JKHL$-dimensional vector of the relationship levels between the retailers and the demand market/country/currency/mode combinations with component ${}^{j}_{kh\hat{l}m}$ denoted by $\eta^{j}_{kh\hat{l}m}$
ρ^{il}_{1jhm}	price associated with the product transacted between manufacturer il and retailer j in currency h via mode m
$\rho^{il}_{1kh\hat{l}}$	price associated with the product transacted between manufacturer il and demand market k in currency h and country $\hat{l}$
$\rho^{j}_{2kh\hat{l}m}$	price associated with the product transacted between retailer j and demand market k in currency h and country $\hat{l}$ via mode m
ρ_3	KHL-dimensional vector of the demand market prices of the product at the demand markets in the currencies and in the countries with component khl denoted by ρ_{3khl}

We now turn to the description of the functions and assume that they are measured in the base currency. We first discuss the production cost, transaction cost, handling, and unit transaction cost functions given in Table 2. Each manufacturer is faced with a certain production cost function that may depend, in general, on the entire vector of production outputs. Furthermore, each manufacturer and each retailer are faced with transaction costs. The transaction costs are affected/influenced by the amount of the product transacted and the relationship levels. As indicated in the introduction, relationship levels affect transaction costs ([13], [44]). This is especially important in international exchanges in which transaction costs may be significant. Hence, the transaction cost functions depend on flows associated with the global supply chain network and the social network.

Table 2: Production, Handling, Transaction, and Unit Transaction Cost Functions

Notation	**Definition**
$f^{il}(q) = f^{il}(Q^1, Q^2)$	the production cost function of manufacturer i in country l
$c_j(Q^1)$	the handling/conversion cost function of retailer j
$c^{il}_{jhm}(q^{il}_{jhm}, \eta^{il}_{jhm})$	the transaction cost function of manufacturer il transacting with retailer j in currency h via mode m
$c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}}, \eta^{il}_{kh\hat{l}})$	the transaction cost function of manufacturer il transacting with demand market $kh\hat{l}$ via the Internet
$\hat{c}^{il}_{jhm}(q^{il}_{jhm}, \eta^{il}_{jhm})$	the transaction cost function of retailer j transacting with manufacturer il in currency h via mode m
$c^{j}_{kh\hat{l}m}(q^{j}_{kh\hat{l}m}, \eta^{j}_{kh\hat{l}m})$	the transaction cost function of retailer j transacting with demand market $kh\hat{l}$ via mode m
$\hat{c}^{il}_{kh\hat{l}}(Q^2, Q^3, \eta^2, \eta^3)$	the unit transaction cost function associated with consumers at demand market $kh\hat{l}$ in obtaining the product from manufacturer il
$\hat{c}^{j}_{kh\hat{l}m}(Q^2, Q^3, \eta^2, \eta^3)$	the unit transaction cost function associated with consumers at demand market $kh\hat{l}$ in obtaining the product from retailer j via mode m

Each retailer is also faced with what we term a *handling/conversion* cost (cf. Table 2), which may include, for example, the cost of handling and storing the product plus the cost associated with transacting in the different currencies. The handling/conversion cost of a retailer is a function of how much he has obtained of the product from the various manufacturers in the different countries and what currency the transactions took place in and in what transaction mode. For the sake of generality, however, we allow the handling functions to depend also on the amounts of the product held and transacted by other retailers.

The consumers at each demand market are faced with a unit transaction cost. As in the case of the manufacturers and the retailers, higher relationship levels may potentially reduce transaction costs, which means that they can lead to quantifiable cost reductions. The unit transaction costs depend on the amounts of the product that the retailers and the manufacturers transact with the demand markets as well as on the vectors of relationships established with the demand markets. The generality of the unit transaction cost function structure enables the modeling of competition on the demand side. Moreover, it allows for information exchange between the consumers at the demand markets who may inform one another as to their relationship levels which, in turn, can affect the transaction costs. We assume that the production cost, the transaction cost, and the handling cost functions are convex and continuously differentiable and that the unit cost functions are continuous.

We now turn to the description of the relationship production cost and relationship value functions and, finally, the risk functions and the demand functions. We assume that the relationship production cost functions as well as the risk functions are convex and continuously differentiable. The relationship value functions are assumed to be concave and continuously differentiable. The demand functions are assumed to be continuous.

Table 3: Relationship Production Cost and Relationship Value Functions

Notation	**Definition**
$b^{il}_{jhm}(\eta^{il}_{jhm})$	the relationship production cost function associated with manufacturer il and retailer jh transacting in mode m
$b^{il}_{kh\hat{l}}(\eta^{il}_{kh\hat{l}})$	the relationship production cost function associated with manufacturer il and demand market $kh\hat{l}$
$\hat{b}^{il}_{jhm}(\eta^{il}_{jhm})$	the relationship production cost function associated with retailer j transacting with manufacturer il in currency h via mode m
$b^{j}_{kh\hat{l}m}(\eta^{j}_{kh\hat{l}m})$	the relationship production cost function associated with retailer j and demand market $kh\hat{l}$ in transacting in mode m
$v^{il}(\eta^1, \eta^2, Q^1, Q^2)$	the relationship value function associated with manufacturer il
$v^{j}(\eta^1, \eta^3, Q^1, Q^3)$	the relationship value function associated with retailer j

We start by describing the relationship production cost and relationship value functions that are given in Table 3. We assume that each manufacturer may actively try to achieve a certain relationship level with a retailer and/or a demand market as proposed in Golicic, Foggin, and

Mentzer [17]. Furthermore, each retailer may actively try to achieve a certain relationship level with a manufacturer and/or demand market. The relationship production function reflects how much money, for example, in the form of gifts and/or additional time or service a manufacturer or retailer has to spend in order to achieve a particular relationship level with a manufacturer, retailer, or demand market. These relationship production cost functions may be distinct for each such combination. Their specific functional forms may be influenced by such factors as the willingness of retailers or demand markets to establish/maintain a relationship as well as the level of previous business relationships and private relationships that exist. Hence, we assume that these production cost functions are also affected and influenced by the relationship levels. Crosby and Stephens [10] indicate that the relationship strength changes with the amount of buyer-seller interaction and communication. In a global setting, cultural differences, difficulties with languages, and distances, may also play a role in making it more costly to establish (and to maintain) a specific relationship level (cf. [22]).

The relationship value functions reflect the fact that the relationship level per se may have some value to the particular decision-maker. As described in the Introduction, multiple authors from different disciplines have highlighted the effects of relationships in economic/business transactions that go beyond the reduction of transaction costs and risks. By explicitly including relationship value, the model captures a spectrum of potential monetary and, hence, quantifiable, effects of relationships.

We now describe the risk functions as presented in Table 4. We note that the risk functions in our model are functions of both the product transactions and the relationship levels. Jüttner, Peck, and Christopher [25] suggest that supply chain-relevant risk sources falls into three categories: environmental risk sources (e.g., fire, social-political actions, or "acts of God"), organizational risk sources (e.g., production uncertainties), and network-related risk sources. Johnson [24] and Norrman and Jansson [38] argue that network-related risk arises from the interaction between organizations within the supply chain, e.g., due to insufficient interaction and cooperation. Here, we model supply chain organizational risk and network-related risk by defining the risk as a function of product flows as well as relationship levels. We use relationship levels (levels of cooperation) as a way of possibly mitigating network-related risk. We also note that by including the exchange rates in our model, in order to convert the prices to the base currency, we are actually mitigating exchange rate risk. Of course, in certain situations; see also Granovetter [18], the risk may be adversely affected by higher levels of relationships. Nevertheless, the functions in Table 4 explicitly include relationship levels and product transactions as inputs into the risk functions and reflect this dependence.

Table 4: Risk Functions

Notation	**Definition**
$r^{il}(Q^1, Q^2, \eta^1, \eta^2)$	the risk incurred by manufacturer il in his transactions
$r^j(Q^1, Q^3, \eta^1, \eta^3)$	the risk incurred by retailer j in his transactions

The demand functions as given in Table 5 are associated with the bottom-tiered nodes of the global supply chain network. The demand of consumers for the product at a demand market in a currency and country depends, in general, not only on the price of the product at that demand market (and currency and country) but also on the prices of the product at the other demand markets (and in other countries and currencies). Consequently, consumers at a demand market, in a sense, also compete with consumers at other demand markets.

Table 5: Demand Functions

Notation	**Definition**
$d_{kh\hat{l}}(\rho_3)$	the demand for the product at demand market k transacted in currency h in country $\hat{l}$ as a function of the demand market price vector

We now turn to describing the behavior of the various economic decision-makers. The model is presented, for ease of exposition, for the case of a single homogeneous product. It can also handle multiple products through a replication of the links and added notation. We first focus on the manufacturers. We then turn to the retailers, and, subsequently, to the consumers at the demand markets.

The Behavior of the Manufacturers

The manufacturers are involved in the production of a homogeneous product and in transacting with the retailers physically or electronically as well as directly with the demand markets electronically. Furthermore, they are also involved in establishing the corresponding relationship levels. The quantity of the product produced by manufacturer il must satisfy the following conservation of flow equation:

$$q^{il} = \sum_{j=1}^{J} \sum_{h=1}^{H} \sum_{m=1}^{2} q^{il}_{jhm} + \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} q^{il}_{kh\hat{l}}, \tag{1}$$

which states that the quantity of the product produced by manufacturer il is equal to the sum of the quantities transacted between the manufacturer and all retailers (via the two modes) and the demand markets. Hence, in view of (1), and as noted in Table 2, we have that for each manufacturer il the production cost $f^{il}(q) = f^{il}(Q^1, Q^2)$. Furthermore, each manufacturer may actively try to achieve a certain relationship level with a retailer and/or a demand market.

Each manufacturer il tries to maximize his profits. He faces total costs that equal the sum of his production cost plus the total transaction costs and the costs that he incurs in establishing and maintaining his relationship levels. His revenue, in turn, is equal to the sum of the price that he can obtain times the exchange rate multiplied by quantities of the product transacted.

Furthermore, each manufacturer tries to minimize his risk and to maximize the relationship value generated by interacting with the other decision-makers. This means that he tries to create a relationship value that is as high as possible taking the other criteria into consideration, subject to his individual weight assignment to this criterion.

The Multicriteria Decision-Making Problem Faced by a Manufacturer

We can now construct the multicriteria decision-making problem facing a manufacturer which allows him to weight the criteria of profit maximization, risk minimization, and relationship value maximization in an individual manner. Manufacturer il's multicriteria decision-making objective function is denoted by U^{il}. Assume that manufacturer il assigns a nonnegative weight α^{il} to the risk generated and a nonnegative weight β^{il} to the relationship value. The weight associated with profit maximization serves as the numeraire and is set equal to 1. The nonnegative weights measure the importance of the risk and the relationship value and, in addition, transform these values into monetary units. Let now ρ^{il*}_{1jhm} denote the actual price charged by manufacturer il for the product in currency h to retailer j transacting via mode m and let $\rho^{il*}_{1kh\hat{l}}$, in turn, denote the actual price associated with manufacturer il transacting electronically with demand market $kh\hat{l}$. We later discuss how such prices are recovered. We can now construct a value function for each manufacturer (cf. [26], [31], [46], and the references therein) using a constant additive weight value function. Therefore, the multicriteria decision-making problem of manufacturer il can be expressed as:

$$\text{Maximize} \quad U^{il} = \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}(\rho_{1jhm}^{il*} \times e_h)q_{jhm}^{il} + \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}(\rho_{1kh\hat{l}}^{il*} \times e_h)q_{kh\hat{l}}^{il} - f^{il}(Q^1, Q^2)$$

$$-\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} c_{jhm}^{il}(q_{jhm}^{il}, \eta_{jhm}^{il}) - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il}, \eta_{kh\hat{l}}^{il}) - \sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2} b_{jhm}^{il}(\eta_{jhm}^{il})$$

$$-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} b_{kh\hat{l}}^{il}(\eta_{kh\hat{l}}^{il}) - \alpha^{il} r^{il}(Q^1, Q^2, \eta^1, \eta^2) + \beta^{il} v^{il}(\eta^1, \eta^2, Q^1, Q^2) \tag{2}$$

subject to:

$$q_{jhm}^{il} \geq 0, \quad q_{kh\hat{l}}^{il} \geq 0, \quad \forall j, h, m, k, \hat{l}, \tag{3}$$

$$0 \leq \eta_{jhm}^{il} \leq 1, \quad 0 \leq \eta_{kh\hat{l}}^{il} \leq 1, \quad \forall j, h, m, k, \hat{l}. \tag{4}$$

The first seven terms on the right-hand side of the equal sign in (2) represent the profit which is to be maximized, the next term represents the weighted risk which is to be minimized and the last term represents the weighted relationship value, which is to be maximized. The relationship values lie in the range between 0 and 1 and, hence, we need (4).

The Optimality Conditions of Manufacturers

Here we assume that the manufacturers compete in a noncooperative fashion following Nash [36, 37]. Hence, each manufacturer seeks to determine his optimal strategies, that is, product transactions, given those of the other manufacturers. The optimality conditions of all manufacturers i; $i = 1, \ldots, I$; in all countries: l; $l = 1, \ldots, L$, simultaneously, under the above assumptions (cf. [2], [16], [29]), can be compactly expressed as: determine $(Q^{1*}, Q^{2*}, \eta^{1*}, \eta^{2*}) \in \mathcal{K}^1$, satisfying

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial f^{il}(Q^{1*}, Q^{2*})}{\partial q_{jhm}^{il}} + \alpha^{il}\frac{\partial r^{il}(Q^{1*}, Q^{2*}, \eta^{1*}, \eta^{2*})}{\partial q_{jhm}^{il}} + \frac{\partial c_{jhm}^{il}(q_{jhm}^{il*}, \eta_{jhm}^{il*})}{\partial q_{jhm}^{il}}\right.$$

$$\left.-\beta^{il}\frac{\partial v^{il}(\eta^{1*}, \eta^{2*}, Q^{1*}, Q^{2*})}{\partial q_{jhm}^{il}} - \rho_{1jhm}^{il*} \times e_h\right] \times \left[q_{jhm}^{il} - q_{jhm}^{il*}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial f^{il}(Q^{1*}, Q^{2*})}{\partial q_{kh\hat{l}}^{il}} + \alpha^{il}\frac{\partial r^{il}(Q^{1*}, Q^{2*}, \eta^{1*}, \eta^{2*})}{\partial q_{kh\hat{l}}^{il}} + \frac{\partial c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il*}, \eta_{kh\hat{l}}^{il*})}{\partial q_{kh\hat{l}}^{il}}\right.$$

$$
-\beta^{il}\frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial q^{il}_{kh\hat{l}}} - \rho^{il*}_{1kh\hat{l}} \times e_h\Bigg] \times \left[q^{il}_{kh\hat{l}} - q^{il*}_{kh\hat{l}}\right]
$$

$$
+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial c^{il}_{jhm}(q^{il*}_{jhm},\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}} + \frac{\partial b^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}} - \beta^{il}\frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial \eta^{il}_{jhm}}\right.
$$

$$
\left.+\alpha^{il}\frac{\partial r^{il}(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})}{\partial \eta^{il}_{jhm}}\right] \times \left[\eta^{il}_{jhm} - \eta^{il*}_{jhm}\right]
$$

$$
+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}},\eta^{il*}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} + \frac{\partial b^{il}_{kh\hat{l}}(\eta^{il*}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \beta^{il}\frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial \eta^{il}_{kh\hat{l}}}\right.
$$

$$
\left.+\alpha^{il}\frac{\partial r^{il}(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})}{\partial \eta^{il}_{kh\hat{l}}}\right] \times \left[\eta^{il}_{kh\hat{l}} - \eta^{il*}_{kh\hat{l}}\right] \geq 0, \quad \forall (Q^1,Q^2,\eta^1,\eta^2) \in \mathcal{K}^1, \tag{5}
$$

where

$$
\mathcal{K}^1 \equiv \left[(Q^1,Q^2,\eta^1,\eta^2)|q^{il}_{jhm} \geq 0,\ q^{il}_{kh\hat{l}} \geq 0,\ 0 \leq \eta^{il}_{jhm} \leq 1,\ 0 \leq \eta^{il}_{kh\hat{l}} \leq 1,\ \forall i,l,j,h,m,k,\hat{l}\right]. \tag{6}
$$

The inequality (5), which is a variational inequality (cf. [29]) has a meaningful economic interpretation. From the first term in (5) we can see that, if there is a positive amount of the product transacted either in a classical manner or via the Internet from a manufacturer to a retailer, then the sum of the marginal production cost, the weighted marginal risk, and the marginal transaction cost must be equal to the weighted marginal relationship value plus the price (times the exchange rate) that the retailer is willing to pay for the product. If the first sum, in turn, exceeds the second one then there will be no product transacted.

The second term in (5) states that there will be a positive flow of the product transacted between a manufacturer and a demand market if the sum of the marginal production cost, the weighted marginal risk, and the marginal cost of transacting via the Internet for the manufacturer with consumers is equal to the weighted marginal relationship value plus the price (times the exchange rate) that the consumers are willing to pay for the product at the demand market.

The third and the fourth term in (5) show that if there is a positive relationship level (and that level is less than one) between a pair of decision-makers then the marginal cost associated with the level is equal to the marginal reduction in transaction costs plus the weighted marginal value of the relationship and the weighted marginal reduction in risk.

The Behavior of the Retailers

The retailers (cf. Figure 1), in turn, are involved in transactions both with the manufacturers in the different countries, as well as with the ultimate consumers associated with the demand markets for the product in different countries and currencies and represented by the bottom tier of nodes in both the global supply chain network and the social network.

As in the case of manufacturers, the retailers have to bear some costs to establish and maintain relationship levels with manufacturers and with the consumers, who are the ultimate purchasers/buyers of the product. Furthermore, the retailers, which can be either physical or virtual, also have associated transaction costs in regards to transacting with the manufacturers, which we assume can be dependent on the type of currency as well as the manufacturer. Retailers also are faced with risk in their transactions. As in the case of the manufacturers, the transaction cost functions and the risk functions depend on the amounts of the product transacted as well as the relationship levels.

Each retailer j tries to maximize profits and relationship values with the manufacturers and the consumers and to minimize his individual risk associated with his transactions with these criteria weighted in an individual fashion.

A Retailer's Multicriteria Decision-Making Problem

Retailer j assigns a nonnegative weight δ^j to his risk and a nonnegative weight γ^j to his relationship value. The weight associated with profit maximization is set equal to 1 and serves as the numeraire (as in the case of the manufacturers). The actual price charged for the product by retailer j is denoted by $\rho^{j*}_{2kh\hat{l}m}$, and is associated with transacting with consumers at demand market k in currency h and country $\hat{l}$ via mode m. Later, we discuss how such prices are arrived at. We are now ready to construct the multicriteria decision-making problem faced by a retailer which combines the individual weights with the criteria of profit maximization, risk minimization, and relationship value maximization. Let U^j denote the multicriteria objective function associated with retailer j with his multicriteria decision-making problem expressed as:

$$\text{Maximize} \quad U^j = \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}(\rho^{j*}_{2kh\hat{l}m} \times e_h)q^j_{kh\hat{l}m} - c_j(Q^1) - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\hat{c}^{il}_{jhm}(q^{il}_{jhm}, \eta^{il}_{jhm})$$

$$- \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} c^j_{kh\hat{l}m}(q^j_{kh\hat{l}m}, \eta^j_{kh\hat{l}m}) - \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} \hat{b}^{il}_{jhm}(\eta^{il}_{jhm}) - \sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} b^j_{kh\hat{l}m}(\eta^j_{kh\hat{l}m})$$

$$- \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} (\rho^{il*}_{1jhm} \times e_h) q^{il}_{jhm} - \delta^j r^j(Q^1, Q^3, \eta^1, \eta^3) + \gamma^j v^j(Q^1, Q^3, \eta^1, \eta^3) \tag{7}$$

subject to:

$$\sum_{k=1}^{K} \sum_{h=1}^{H} \sum_{\hat{l}=1}^{L} \sum_{m=1}^{2} q^j_{kh\hat{l}m} \leq \sum_{i=1}^{I} \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} q^{il}_{jhm}, \tag{8}$$

$$q^{il}_{jhm} \geq 0, \quad q^j_{kh\hat{l}m} \geq 0, \quad \forall i, l, k, h, \hat{l}, m, \tag{9}$$

$$0 \leq \eta^{il}_{jhm} \leq 1, \quad 0 \leq \eta^j_{kh\hat{l}m} \leq 1, \quad \forall i, l, h, m, k, \hat{l}. \tag{10}$$

The first seven terms on the right-hand side of the equal sign in (7) represent the profit which is to be maximized, the next term represents the weighted risk which is to be minimized and the last term represents the weighted relationship value, which is to be maximized. Constraint (8) states that consumers cannot purchase more of the product from a retailer than is held "in stock."

The Optimality Conditions of Retailers

We now turn to the optimality conditions of the retailers. Each retailer faces the multicriteria decision-making problem (7), subject to (8), the nonnegativity assumption on the variables (9), and the assumptions for the relationship values (10). As in the case of the manufacturers, we assume that the retailers compete in a noncooperative manner, given the actions of the other retailers. Retailers seek to determine the optimal transactions associated with the demand markets and with the manufacturers. In equilibrium, all the transactions between the tiers of the decision-makers will have to coincide, as we will see later in this section.

Since we have assumed that the handling, transaction cost, and risk functions are continuously differentiable and convex, and that the relationship values are also continuously differentiable but concave, the optimality conditions for all the retailers satisfy the variational inequality: determine $(Q^{1*}, Q^{3*}, \eta^{1*}, \eta^{3*}, \lambda^*) \in \mathcal{K}^2$, such that

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{l=}^{L} \sum_{h=1}^{H} \sum_{m=1}^{2} \left[\delta^j \frac{\partial r^j(Q^{1*}, Q^{3*}, \eta^{1*}, \eta^{3*})}{\partial q^{il}_{jhm}} + \frac{\partial c_j(Q^{1*})}{\partial q^{il}_{jhm}} + \rho^{il*}_{1jhm} \times e_h + \frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm}, \eta^{il*}_{jhm})}{\partial q^{il}_{jhm}} \right.$$

$$-\gamma^j \frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^{il}_{jhm}} - \lambda^*_j\Bigg] \times \left[q^{il}_{jhm} - q^{il*}_{jhm}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\Bigg[\delta^j \frac{\partial r^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^{j}_{kh\hat{l}m}} + \frac{\partial c^j_{kh\hat{l}m}(q^{j*}_{kh\hat{l}m},\eta^{j*}_{kh\hat{l}m})}{\partial q^{j}_{kh\hat{l}m}} - \gamma^j \frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^{j}_{kh\hat{l}m}}$$

$$-\rho^{j*}_{2kh\hat{l}m} \times e_h + \lambda^*_j\Bigg] \times \left[q^{j}_{kh\hat{l}m} - q^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{l=}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}\Bigg[\delta^j \frac{\partial r^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial \eta^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm},\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}} - \gamma^j \frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial \eta^{il}_{jhm}}$$

$$+\frac{\partial \hat{b}^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}\Bigg] \times \left[\eta^{il}_{jhm} - \eta^{il*}_{jhm}\right]$$

$$+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\Bigg[\delta^j \frac{\partial r^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial \eta^{j}_{kh\hat{l}m}} + \frac{\partial c^j_{kh\hat{l}m}(q^{j*}_{kh\hat{l}m},\eta^{j*}_{kh\hat{l}m})}{\partial \eta^{j}_{kh\hat{l}m}} - \gamma^j \frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial \eta^{j}_{kh\hat{l}m}}$$

$$+\frac{\partial b^{j}_{khl}(\eta^{j*}_{kh\hat{l}m})}{\partial \eta^{j}_{kh\hat{l}m}}\Bigg] \times \left[\eta^{j}_{kh\hat{l}m} - \eta^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q^{il*}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} q^{j*}_{kh\hat{l}m}\right] \times \left[\lambda_j - \lambda^*_j\right] \geq 0, \quad \forall (Q^1,Q^3,\eta^1,\eta^3,\lambda) \in \mathcal{K}^2, \tag{11}$$

where

$$\mathcal{K}^2 \equiv \left[(Q^1,Q^3,\eta^1,\eta^3,\lambda)|q^{il}_{jhm} \geq 0,\ q^{j}_{kh\hat{l}m} \geq 0,\ 0 \leq \eta^{il}_{jhm} \leq 1,\ 0 \leq \eta^{j}_{kh\hat{l}m} \leq 1,\ \lambda_j \geq 0,\ \forall i,l,j,h,m,k,\hat{l}\right]. \tag{12}$$

Here λ_j denotes the Lagrange multiplier associated with constraint (8) and λ is the column vector of all the retailers' Lagrange multipliers. These Lagrange multipliers can also be interpreted as shadow prices. Indeed, according to the fifth term in (11), λ^*_j serves as the price to "clear the market" at retailer j.

The economic interpretation of the retailers' optimality conditions is very interesting. The first term in (11) states that if there is a positive amount of product transacted between a manufacturer/retailer pair via mode m and currency h, that is, $q^{il*}_{jhm} > 0$, then the shadow price at the retailer, λ^*_j, plus the weighted marginal relationship value is equal to the price charged for the product (times the exchange rate) plus the various marginal costs and the associated weighted marginal risk. In addition, the second term in (11) shows that, if consumers at demand market $kh\hat{l}$ purchase the product from a particular retailer j transacted through mode m, which means that, if $q^{j*}_{kh\hat{l}m}$ is positive, then the price charged by retailer j, $\rho^{j*}_{2kh\hat{l}m}$ (times the exchange rate), plus the weighted marginal relationship value, is equal to λ^*_j plus the marginal transaction costs in dealing with the demand market and the weighted marginal costs for the risk that he has to bear. One also obtains interpretations from (11) as to the economic conditions at which the relationship levels associated with retailers interacting with either the manufacturers or the demand markets will take on positive values.

The Consumers at the Demand Markets

We now describe the consumers located at the demand markets. The consumers can transact through physical and electronic links with the retailers and through electronic links with the manufacturers. The consumers at demand market k in country $\hat{l}$ take into account the price charged for the product transacted in currency h and via mode m by retailer j, which is denoted by $\rho^{j*}_{2kh\hat{l}m}$, the price charged by manufacturer il, which was denoted by $\rho^{il*}_{1kh\hat{l}}$, and the exchange rate, plus the transaction costs, in making their consumption decisions. The equilibrium conditions for demand market $kh\hat{l}$, thus, take the form: for all retailers: $j = 1, \dots, J$, demand markets k; $k = 1, \dots, K$; currencies: h; $h = 1, \dots, H$, and modes m; $m = 1, 2$:

$$\rho^{j*}_{2kh\hat{l}m} \times e_h + \hat{c}^j_{kh\hat{l}m}(Q^{2*}, Q^{3*}, \eta^{2*}, \eta^{3*}) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{j*}_{kh\hat{l}m} > 0, \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{j*}_{kh\hat{l}m} = 0, \end{cases} \tag{13}$$

and for all manufacturers il; $i = 1, \dots, I$ and $l = 1, \dots, L$:

$$\rho^{il*}_{1kh\hat{l}} \times e_h + \hat{c}^{il}_{kh\hat{l}}(Q^{2*}, Q^{3*}, \eta^{2*}, \eta^{3*}) \begin{cases} = \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{il*}_{kh\hat{l}} > 0, \\ \geq \rho^*_{3kh\hat{l}}, & \text{if} \quad q^{il*}_{kh\hat{l}} = 0. \end{cases} \tag{14}$$

In addition, we must have that for all $k, h, \hat{l}$:

$$d_{kh\hat{l}}(\rho^*_3) \begin{cases} = \sum_{j=1}^{J} \sum_{m=1}^{2} q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} q^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} > 0, \\ \leq \sum_{j=1}^{J} \sum_{m=1}^{2} q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I} \sum_{l=1}^{L} q^{il*}_{kh\hat{l}}, & \text{if} \quad \rho^*_{3kh\hat{l}} = 0. \end{cases} \tag{15}$$

Conditions (13) state that consumers at demand market $kh\hat{l}$ will purchase the product from retailer j via mode m, if the price charged by the retailer for the product times the exchange rate plus the transaction cost (from the perspective of the consumer) does not exceed the price that the consumers are willing to pay for the product in that currency and country, i.e., $\rho^*_{3kh\hat{l}}$. Note that, according to (13), if the transaction costs are identically equal to zero, then the price faced by the consumers for a given product is the price charged by the retailer for the particular product and currency in the country plus the rate of appreciation in the currency. Condition (14) states the analogue, but for the case of electronic transactions with the manufacturers.

Condition (15), on the other hand, states that, if the price the consumers are willing to pay for the product at a demand market is positive, then the quantity of the product at the demand market is precisely equal to the demand.

Note that, according to (13) – (15) we assume that there is a single price for the product at a demand market in a country associated with *either* mode of transaction in a currency. In other words, we assume that consumers are price-sensitive and choose to transact a particular volume with either mode (or combination) of transaction, provided that the price at the demand market is minimal.

In equilibrium, conditions (13), (14), and (15) will have to hold for all demand markets and these, in turn, can be expressed also as an inequality analogous to those in (5) and (11) and given by: determine $(Q^{2*}, Q^{3*}, \rho^*_3) \in R_+^{(IL+2J+1)KHL}$, such that

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\rho^{j*}_{2kh\hat{l}m} \times e_h + \hat{c}^{j}_{kh\hat{l}m}(Q^{2*}, Q^{3*}, \eta^{2*}, \eta^{3*}) - \rho^*_{3kh\hat{l}}\right] \times \left[q^{j}_{kh\hat{l}m} - q^{j*}_{kh\hat{l}m}\right]$$

$$+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\rho^{il*}_{1kh\hat{l}} \times e_h + \hat{c}^{il}_{kh\hat{l}}(Q^{2*}, Q^{3*}, \eta^{2*}, \eta^{3*}) - \rho^*_{3kh\hat{l}}\right] \times \left[q^{il}_{kh\hat{l}} - q^{il*}_{kh\hat{l}}\right]$$

$$+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2} q^{j*}_{kh\hat{l}m} + \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il*}_{kh\hat{l}} - d_{kh\hat{l}}(\rho^*_3)\right] \times \left[\rho_{3kh\hat{l}} - \rho^*_{3kh\hat{l}}\right] \geq 0,$$

$$\forall (Q^2, Q^3, \rho_3) \in R_+^{(IL+2J+1)KHL}. \tag{16}$$

In the context of the consumption decisions, we have utilized demand functions, whereas profit functions, which correspond to objective functions, were used in the case of the manufacturers and the retailers. Since we can expect the number of consumers to be much greater than that of the manufacturers and retailers we believe that such a formulation is more natural. Also, note that the relationship levels in (16) are assumed as given. They are endogenous to the integrated model as is soon revealed.

The Equilibrium Conditions of the Supernetwork

In equilibrium, the product flows that the manufacturers in different countries transact with the retailers must coincide with those that the retailers actually accept from them. In addition, the amounts of the product that are obtained by the consumers in the different countries and currencies must be equal to the amounts that both the manufacturers and the retailers actually provide. Hence, although there may be competition between decision-makers at the same level of tier of nodes of the supernetwork there must be cooperation between decision-makers associated with pairs of nodes. Thus, in equilibrium, the prices and product transactions must satisfy the sum of the optimality conditions (5) and (11) and the equilibrium conditions (16). We make these statements rigorous through the subsequent definition and variational inequality derivation.

Definition 1: Supernetwork Equilibrium

The equilibrium state of the supernetwork is one where the product transactions and relationship levels between the tiers of the supernetwork coincide and the product transactions, relationship levels, and prices satisfy the sum of conditions (5), (11), and (16).

The equilibrium state is equivalent to the following:

Theorem 1: Variational Inequality Formulation

The equilibrium conditions governing the supernetwork model according to Definition 1 are equivalent to the solution of the variational inequality given by: determine $(Q^{1*}, Q^{2*}, Q^{3*}, \eta^{1*}, \eta^{2*}, \eta^{3*}, \lambda^*, \rho_3^*) \in \mathcal{K}$, *satisfying:*

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial f^{il}(Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}} + \alpha^{il}\frac{\partial r^{il}(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})}{\partial q^{il}_{jhm}} + \delta^{j}\frac{\partial r^{j}(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^{il}_{jhm}}\right.$$
$$+\frac{\partial c_j(Q^{1*})}{\partial q^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(q^{il*}_{jhm},\eta^{il*}_{jhm})}{\partial q^{il}_{jhm}} + \frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm},\eta^{il*}_{jhm})}{\partial q^{il}_{jhm}} - \beta^{il}\frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}}$$
$$\left.-\gamma^{j}\frac{\partial v^{j}(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^{il}_{jhm}} - \lambda^*_j\right]\times\left[q^{il}_{jhm} - q^{il*}_{jhm}\right]$$

$$
\begin{aligned}
&+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial f^{il}(Q^{1*},Q^{2*})}{\partial q^{il}_{kh\hat{l}}}+\alpha^{il}\frac{\partial r^{il}(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})}{\partial q^{il}_{kh\hat{l}}}+\frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}},\eta^{il*}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}}\right.\\
&\qquad\left.-\beta^{il}\frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial q^{il}_{kh\hat{l}}}+\hat{c}^{il}_{kh\hat{l}}(Q^{2*},Q^{3*},\eta^{2*},\eta^{3*})-\rho^*_{3kh\hat{l}}\right]\times\left[q^{il}_{kh\hat{l}}-q^{il*}_{kh\hat{l}}\right]\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\delta^j\frac{\partial r^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^j_{kh\hat{l}m}}+\frac{\partial c^j_{kh\hat{l}m}(q^{j*}_{kh\hat{l}m},\eta^{j*}_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}}+\hat{c}^j_{kh\hat{l}m}(Q^{2*},Q^{3*},\eta^{2*},\eta^{3*})\right.\\
&\qquad\left.+\lambda^*_j-\gamma^j\frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^j_{kh\hat{l}m}}-\rho^*_{3kh\hat{l}}\right]\times\left[q^j_{kh\hat{l}m}-q^{j*}_{kh\hat{l}m}\right]\\
&+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{h=1}^{H}\sum_{m=1}^{2}\left[\frac{\partial c^{il}_{jhm}(q^{il*}_{jhm},\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}+\frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm},\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}+\frac{\partial b^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}+\frac{\partial \hat{b}^{il}_{jhm}(\eta^{il*}_{jhm})}{\partial \eta^{il}_{jhm}}\right.\\
&\qquad+\alpha^{il}\frac{\partial r^{il}(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})}{\partial \eta^{il}_{jhm}}+\delta^j\frac{\partial r^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial \eta^{il}_{jhm}}-\beta^{il}\frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial \eta^{il}_{jhm}}\\
&\qquad\left.-\gamma^j\frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial \eta^{il}_{jhm}}\right]\times\left[\eta^{il}_{jhm}-\eta^{il*}_{jhm}\right]\\
&+\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}},\eta^{il*}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}}+\frac{\partial b^{il}_{kh\hat{l}}(\eta^{il*}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}}+\alpha^{il}\frac{\partial r^{il}(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})}{\partial \eta^{il}_{kh\hat{l}}}\right.\\
&\qquad\left.-\beta^{il}\frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial \eta^{il}_{kh\hat{l}}}\right]\times\left[\eta^{il}_{kh\hat{l}}-\eta^{il*}_{kh\hat{l}}\right]\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}\left[\delta^j\frac{\partial r^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial \eta^j_{kh\hat{l}m}}+\frac{\partial c^j_{kh\hat{l}m}(q^{j*}_{kh\hat{l}m},\eta^{j*}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}}+\frac{\partial b^j_{khl}(\eta^{j*}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}}\right.\\
&\qquad\left.-\gamma^j\frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial \eta^j_{kh\hat{l}m}}\right]\times\left[\eta^j_{kh\hat{l}m}-\eta^{j*}_{kh\hat{l}m}\right]\\
&+\sum_{j=1}^{J}\left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2}q^{il*}_{jhm}-\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2}q^{j*}_{kh\hat{l}m}\right]\times\left[\lambda_j-\lambda^*_j\right]\\
&+\sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\left[\sum_{j=1}^{J}\sum_{m=1}^{2}q^{j*}_{kh\hat{l}m}+\sum_{i=1}^{I}\sum_{l=1}^{L}q^{il*}_{kh\hat{l}}-d_{kh\hat{l}}(\rho^*_3)\right]\times\left[\rho_{3kh\hat{l}}-\rho^*_{3kh\hat{l}}\right]\geq 0,\\
&\qquad\forall(Q^1,Q^2,Q^3,\eta^1,\eta^2,\eta^3,\lambda,\rho_3)\in\mathcal{K}, \qquad (17)
\end{aligned}
$$

where

$$\mathcal{K} \equiv \Big[(Q^1, Q^2, Q^3, \eta^1, \eta^2, \eta^3, \lambda, \rho_3) | q^{il}_{jhm} \geq 0,\ q^{il}_{kh\hat{l}} \geq 0,\ q^{j}_{kh\hat{l}m} \geq 0,\ 0 \leq \eta^{il}_{jhm} \leq 1,\ 0 \leq \eta^{il}_{kh\hat{l}} \leq 1,$$

$$0 \leq \eta^{j}_{kh\hat{l}m} \leq 1,\ \rho_{3kh\hat{l}} \geq 0,\ \lambda_j \geq 0, \quad \forall i, l, j, h, m, k, \hat{l}\Big]. \tag{18}$$

Proof: Summation of inequalities (5), (11), and (16), yields, after algebraic simplification, the variational inequality (17). We now establish the converse, that is, that a solution to variational inequality (17) satisfies the sum of conditions (5), (11), and (16) and is, hence, an equilibrium according to Definition 1. To inequality (17) add the term $+\rho^{il*}_{1jhm} \times e_h$ - $\rho^{il*}_{1jhm} \times e_h$ to the first set of brackets preceding the multiplication sign. Similarly, add the term $+\rho^{il*}_{1kh\hat{l}} \times e_h$ - $\rho^{il*}_{1kh\hat{l}} \times e_h$ to the term in brackets preceding the second multiplication sign. Finally, add the term $+\rho^{j*}_{2kh\hat{l}m} \times e_h$ - $\rho^{j*}_{2kh\hat{l}m} \times e_h$ to the term preceding the third multiplication sign in (17). The addition of such terms does not alter (17) since the value of these terms is zero. The resulting inequality can be rewritten to become equivalent to the price and material flow pattern satisfying the sum of the conditions (5), (11), and (16). The proof is complete. $\square$

We now put variational inequality (17) into standard form which will be utilized in the subsequent sections. For additional background on variational inequalities and their applications, see the book by Nagurney [29]. In particular, we have that variational inequality (17) can be expressed as:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{19}$$

where $X \equiv (Q^1, Q^2, Q^3, \eta^1, \eta^2, \eta^3, \lambda, \rho_3)$ and $F(X) \equiv (F_{iljhm}, F_{ilkh\hat{l}}, F_{jkh\hat{l}m}, \hat{F}_{iljhm}, \hat{F}_{ilkh\hat{l}}, \hat{F}_{jkh\hat{l}m}, F_j, F_{kh\hat{l}})$ with indices: $i = 1, \ldots, I$; $l = 1, \ldots, L; j = 1, \ldots, J$; $h = 1, \ldots, H$; $\hat{l} = 1, \ldots, L; m = 1, 2,$ and the specific components of F given by the functional terms preceding the multiplication signs in (17), respectively. The term $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

We now describe how to recover the prices associated with the first two tiers of nodes in the global supply chain network. Such a pricing mechanism guarantees that the optimality conditions (5) and (11) as well as the equilibrium conditions (16) are satisfied individually through the solution of variational inequality (17). Hence, through financial engineering, we are able to integrate both the global supply chain network as well as the social network and to handle both the decision-makers' competitive as well as cooperative behaviors. Indeed, without the latter a supply chain network cannot exist.

Clearly, the components of the vector ρ_3^* are obtained directly from the solution of variational inequality (17) as will be demonstrated explicitly through several numerical examples in Section 5. In order to recover the second tier prices associated with the retailers and the appreciation/exchange rates one can (after solving variational inequality (17) for the particular numerical problem) *either* (cf. (13) or (16)) set $\rho^{j*}_{2kh\hat{l}m} \times e_h = \left[\rho^*_{3kh\hat{l}} - \hat{c}^j_{kh\hat{l}m}(Q^{2*}, Q^{3*}, \eta^{2*}, \eta^{3*})\right]$, for any $j, k, h, \hat{l}, m$ such that $q^{j*}_{kh\hat{l}m} > 0$, *or* (cf. (11)) for any $q^{j*}_{kh\hat{l}m} > 0$, set $\rho^{j*}_{2kh\hat{l}m} \times e_h = \left[\delta^j \frac{\partial r^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^j_{kh\hat{l}m}} + \frac{\partial c^j_{kh\hat{l}m}(q^{j*}_{kh\hat{l}m},\eta^{j*}_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} - \gamma^j \frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^j_{kh\hat{l}m}} + \lambda^*_j\right]$.

Similarly, from (5) we can infer that the top tier prices can be recovered (once the variational inequality (17) is solved with particular data) thus: for any i, l, j, h, m, such that $q^{il*}_{jhm} > 0$, set $\rho^{il*}_{1jhm} \times e_h = \left[\frac{\partial f^{il}(Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}} + \alpha^{il} \frac{\partial r^{il}(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})}{\partial q^{il}_{jhm}} + \frac{\partial c^{il}_{jhm}(q^{il*}_{jhm},\eta^{il*}_{jhm})}{\partial q^{il}_{jhm}} - \beta^{il} \frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial q^{il}_{jhm}}\right]$, or, equivalently, (cf. (11)), to $\left[\lambda^*_j + \gamma^j \frac{\partial v^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^{il}_{jhm}} - \delta^j \frac{\partial r^j(Q^{1*},Q^{3*},\eta^{1*},\eta^{3*})}{\partial q^{il}_{jhm}} - \frac{\partial c_j(Q^{1*})}{\partial q^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il*}_{jhm},\eta^{il*}_{jhm})}{\partial q^{il}_{jhm}}\right]$.

In addition, in order to recover the first tier prices associated with the demand market and the appreciation/exchange rates one can (after solving variational inequality (17) for the particular numerical problem) *either* (cf. (5)) for any $i, l, k, h, \hat{l}$ such that $q^{il*}_{kh\hat{l}} > 0$, set $\rho^{il*}_{1kh\hat{l}} \times e_h = \left[\frac{\partial f^{il}(Q^{1*},Q^{2*})}{\partial q^{il}_{kh\hat{l}}} + \alpha^{il} \frac{\partial r^{il}(Q^{1*},Q^{2*},\eta^{1*},\eta^{2*})}{\partial q^{il}_{kh\hat{l}}} + \frac{\partial c^{il}_{kh\hat{l}}(q^{il*}_{kh\hat{l}},\eta^{il*}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} - \beta^{il} \frac{\partial v^{il}(\eta^{1*},\eta^{2*},Q^{1*},Q^{2*})}{\partial q^{il}_{kh\hat{l}}}\right]$, *or* (cf. (14)) for any $q^{il*}_{kh\hat{l}} > 0$, set $\rho^{il*}_{1kh\hat{l}} \times e_h = \left[\rho^*_{3kh\hat{l}} - \hat{c}^{il}_{kh\hat{l}}(Q^{2*}, Q^{3*}, \eta^{2*}, \eta^{3*})\right]$.

Under the above pricing mechanism, which provides a valuable financial engineering tool, the optimality conditions (5) and (11) as well as the equilibrium conditions (16) also hold separately (as well as for each individual decision-maker) (see also, e.g., [11], [30], [32]).

Note that, if the equilibrium values of the flows (be they product or relationship levels) on links are identically equal to zero, then those links can effectively be removed from the supernetwork (in equilibrium). Moreover, the size of the equilibrium flows represent the "strength" of the respective links. Thus, the supernetwork model developed here also provides us with the emergent integrated social and global supply chain network structures. In addition, the solution of the model reveals the true network structure in terms of the optimal relationships (and their sizes) as well as the optimal product transactions, and the associated prices. In the next Section, we discuss the dynamic evolution of the global product transactions, relationship levels, and prices until an equilibrium is achieved.

Remark

We note that manufacturers as well as distributors and even retailers may be faced with capacity constraints. Capacity limitations can be handled in the above model since the production cost functions, as well as the transaction cost functions and the handling cost functions can assume nonlinear forms (as is standard in the case of modeling capacities on roads in congested urban transportation networks (cf. Sheffi [42]). Of course, one can also impose explicit capacity constraints and this would then just change the underlying feasible set(s) so that $\mathcal{K}$ would need to be redefined accordingly. However, the function $F(X)$ in variational inequality (19) would remain the same (see, e.g., [29]). Also, we emphasize that unit taxes can also be handled by the above model (and its dynamic variant presented in Section 3) by inclusion in the corresponding unit transaction cost functions (cf. (13) and (14)). Finally, since we consider a single homogeneous product the exchange rates e_h are assumed fixed (and relative to a base currency). Once can, of course, investigate numerous exchange rate and demand scenarios by altering the demand functions and the fixed exchange rates and then recomputing the new equilibrium product transaction, price, and relationship level equilibrium patterns.

3. The Dynamic Model

In this Section, we describe the dynamics associated with the integrated global supply chain/social supernetwork model developed in Section 2 and we formulate the corresponding dynamic model as a projected dynamical system (cf. [29], [30], [31], [46]). Importantly, the set of stationary points of the projected dynamical system which describes the dynamic adjustment processes of the global product transactions, relationship levels, and prices, will coincide with the set of solutions to the variational inequality problem (17). In particular, we propose the disequilibrium dynamics of the global product transactions, the relationship levels, as well as the prices until the equilibrium pattern is attained. The dynamics further illuminate the cooperative aspects of the decision-making behaviors that must take place. The dynamics of the product transactions take place on the corresponding links of the global supply chain network whereas the dynamics of the relationship levels take place on the social network component of the supernetwork in Figure 1 until an equilibrium is reached.

The Dynamics of the Product Transactions between the Manufacturers and the Retailers

The dynamics of the product transactions between manufacturers in the countries and the retailers in the different currencies and modes are now described. Note that in order for a transaction between nodes in these two tiers to take place there must be agreement between the pair of decision-makers. Towards that end, we let $\dot{q}^{il}_{jhm}$ denote the rate of change of the product transaction between manufacturer il and retailer j transacting via mode m and currency h and mathematically express it in the following way: for all i, l, j, h, m:

$$\dot{q}^{il}_{jhm} = \begin{cases} \lambda_j + \beta^{il}\frac{\partial v^{il}(\eta^1,\eta^2,Q^1,Q^2)}{\partial q^{il}_{jhm}} + \gamma^j\frac{\partial v^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial q^{il}_{jhm}} - \frac{\partial f^{il}(Q^1,Q^2)}{\partial q^{il}_{jhm}} - \alpha^{il}\frac{\partial r^{il}(Q^1,Q^2,\eta^1,\eta^2)}{\partial q^{il}_{jhm}} \\ -\delta^j\frac{\partial r^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial q^{il}_{jhm}} - \frac{\partial c_j(Q^1)}{\partial q^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(q^{il}_{jhm},\eta^{il}_{jhm})}{\partial q^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm},\eta^{il}_{jhm})}{\partial q^{il}_{jhm}}, \text{ if } q^{il}_{jhm} > 0, \\ \max\{0, \lambda_j + \beta^{il}\frac{\partial v^{il}(\eta^1,\eta^2,Q^1,Q^2)}{\partial q^{il}_{jhm}} + \gamma^j\frac{\partial v^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial q^{il}_{jhm}} - \frac{\partial f^{il}(Q^1,Q^2)}{\partial q^{il}_{jhm}} - \alpha^{il}\frac{\partial r^{il}(Q^1,Q^2,\eta^1,\eta^2)}{\partial q^{il}_{jhm}} \\ -\delta^j\frac{\partial r^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial q^{il}_{jhm}} - \frac{\partial c_j(Q^1)}{\partial q^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(q^{il}_{jhm},\eta^{il}_{jhm})}{\partial q^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm},\eta^{il}_{jhm})}{\partial q^{il}_{jhm}}\}, \text{ if } q^{il}_{jhm} = 0. \end{cases} \tag{20}$$

Hence, the transaction between a manufacturer in a country and a retailer via a mode and in a currency will increase if the price that the retailer is willing to pay the manufacturer plus the weighted marginal relationship values exceed the various marginal costs plus the weighted marginal risks. Moreover, we guarantee that such a transaction never becomes negative through the projection operation (cf. (20)).

The Dynamics of the Product Transactions between the Manufacturers and the Demand Markets

The rate of change of the product transacted between a manufacturer in a country and a demand market/currency/country pair is assumed to be equal to the price the consumers are willing to pay plus the weighted marginal relationship value minus the various costs, including marginal ones, that the manufacturer incurs when transacting with the demand market in a country and currency and the weighted marginal risk. We denote this rate of change by $\dot{q}^{il}_{kh\hat{l}}$, and mathematically, express it in the following way: for all $i, l, k, h, \hat{l}$:

$$\dot{q}^{il}_{kh\hat{l}} = \begin{cases} \rho_{3kh\hat{l}} + \beta^{il} \frac{\partial v^{il}(\eta^1,\eta^2,Q^1,Q^2)}{\partial q^{il}_{kh\hat{l}}} - \frac{\partial f^{il}(Q^1,Q^2)}{\partial q^{il}_{kh\hat{l}}} - \alpha^{il} \frac{\partial r^{il}(Q^1,Q^2,\eta^1,\eta^2)}{\partial q^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}},\eta^{il}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} \\ -\hat{c}^{il}_{kh\hat{l}}(Q^2,Q^3,\eta^2,\eta^3), \text{ if } q^{il}_{kh\hat{l}} > 0, \\ \max\{0, \rho_{3kh\hat{l}} + \beta^{il} \frac{\partial v^{il}(\eta^1,\eta^2,Q^1,Q^2)}{\partial q^{il}_{kh\hat{l}}} - \frac{\partial f^{il}(Q^1,Q^2)}{\partial q^{il}_{kh\hat{l}}} - \alpha^{il} \frac{\partial r^{il}(Q^1,Q^2,\eta^1,\eta^2)}{\partial q^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}},\eta^{il}_{kh\hat{l}})}{\partial q^{il}_{kh\hat{l}}} \\ -\hat{c}^{il}_{kh\hat{l}}(Q^2,Q^3,\eta^2,\eta^3)\}, \text{ if } q^{il}_{kh\hat{l}} = 0. \end{cases} \tag{21}$$

Note that (21) guarantees that the volume of product transacted will not take on a negative value.

The Dynamics of the Product Transactions between the Retailers and the Demand Markets

The rate of change of the product transaction $q^j_{kh\hat{l}m}$, denoted by $\dot{q}^j_{kh\hat{l}m}$, is assumed to be equal to the price the consumers are willing to pay for the product at the demand market plus the weighted marginal relationship value minus the price charged and the various transaction costs and the weighted marginal risk associated with the transaction. Here we also guarantee that the product transactions do not become negative. Hence, we may write: for every $j, k, h, \hat{l}, m$:

$$\dot{q}^j_{kh\hat{l}m} = \begin{cases} \rho_{3kh\hat{l}} + \gamma^j \frac{\partial v^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial q^j_{kh\hat{l}m}} - \delta^j \frac{\partial r^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial q^j_{kh\hat{l}m}} - \frac{\partial c^j_{kh\hat{l}m}(q^j_{kh\hat{l}m},\eta^j_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} \\ -\hat{c}^j_{kh\hat{l}m}(Q^2,Q^3,\eta^2,\eta^3) - \lambda_j, \text{ if } q^j_{kh\hat{l}m} > 0, \\ \max\{0, \rho_{3kh\hat{l}} + \gamma^j \frac{\partial v^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial q^j_{kh\hat{l}m}} - \delta^j \frac{\partial r^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial q^j_{kh\hat{l}m}} - \frac{\partial c^j_{kh\hat{l}m}(q^j_{kh\hat{l}m},\eta^j_{kh\hat{l}m})}{\partial q^j_{kh\hat{l}m}} \\ -\hat{c}^j_{kh\hat{l}m}(Q^2,Q^3,\eta^2,\eta^3) - \lambda_j\}, \text{ if } q^j_{kh\hat{l}m} = 0. \end{cases} \tag{22}$$

According to (22), if the price that the consumers are willing to pay for the product (in the currency and country) plus the weighted marginal relationship value exceed the price that the retailer charges plus the various transaction costs and weighted marginal risk, then the volume of flow of the product to that demand market will increase; otherwise, it will decrease (or remain unchanged).

The Dynamics of the Relationship Levels between the Manufacturers and the Retailers

Now the dynamics of the relationship levels between the manufacturers in the various countries and the retailers are described. The rate of change of the relationship level η^{il}_{jhm}, denoted by $\dot{\eta}^{il}_{jhm}$, is assumed to be equal to the difference between the weighted relationship value for manufacturer il, retailer j, currency h and mode m, and the sum of the marginal costs and the weighted marginal risks. Again, one must also guarantee that the relationship levels do not become negative. Moreover, they may not exceed the level one. Hence, we can immediately write:

$$\dot{\eta}^{il}_{jhm} = \begin{cases} \beta^{il}\frac{\partial v^{il}(\eta^1,\eta^2,Q^1,Q^2)}{\partial \eta^{il}_{jhm}} + \gamma^j \frac{\partial v^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial \eta^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(q^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} \\ -\frac{\partial b^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \frac{\partial \hat{b}^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \alpha^{il}\frac{\partial r^{il}(Q^1,Q^2,\eta^1,\eta^2)}{\partial \eta^{il}_{jhm}} - \delta^j \frac{\partial r^j(Q^{1*},Q^3,\eta^1,\eta^3)}{\partial \eta^{il}_{jhm}}, \text{ if } 0 < \eta^{il}_{jhm} < 1, \\ \min\{1, \max\{0, \beta^{il}\frac{\partial v^{il}(\eta^1,\eta^2,Q^1,Q^2)}{\partial \eta^{il}_{jhm}} + \gamma^j \frac{\partial v^j(Q^1,Q^3,\eta^1,\eta^3)}{\partial \eta^{il}_{jhm}} - \frac{\partial c^{il}_{jhm}(q^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \frac{\partial \hat{c}^{il}_{jhm}(q^{il}_{jhm},\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} \\ -\frac{\partial b^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \frac{\partial \hat{b}^{il}_{jhm}(\eta^{il}_{jhm})}{\partial \eta^{il}_{jhm}} - \alpha^{il}\frac{\partial r^{il}(Q^1,Q^2,\eta^1,\eta^2)}{\partial \eta^{il}_{jhm}} - \delta^j \frac{\partial r^j(Q^{1*},Q^3,\eta^1,\eta^3)}{\partial \eta^{il}_{jhm}}\}\}, \text{ otherwise,} \end{cases} \tag{23}$$

where $\dot{\eta}^{il}_{jhm}$ denotes the rate of change of the relationship level η^{il}_{jhm}.

This shows that if the sum of the weighted relationship values for the manufacturer and the retailer are higher than the total marginal costs plus the total weighted marginal risk, then the level of relationship between that manufacturer and the retailer pair will increase; if they are lower, then the relationship value will decrease.

The Dynamics of the Relationship Levels between the Manufacturers and the Demand Markets

We now describe the dynamics of the relationship levels between the manufacturers and the demand markets. The rate of change of the relationship level $\eta^{il}_{kh\hat{l}}$ responds to the difference between the weighted relationship value for manufacturer il and the sum of the marginal costs and weighted marginal risks. One also must guarantee that these relationship levels do not become negative (nor higher than one). Hence, one may write:

$$\dot{\eta}^{il}_{kh\hat{l}} = \begin{cases} \beta^{il}\frac{\partial v^{il}(\eta^1,\eta^2,Q^1,Q^2)}{\partial \eta^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}},\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \frac{\partial b^{il}_{kh\hat{l}}(\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \alpha^{il}\frac{\partial r^{il}(Q^1,Q^2,\eta^1,\eta^2)}{\partial \eta^{il}_{kh\hat{l}}}, \text{ if } 0 < \eta^{il}_{kh\hat{l}} < 1, \\ \min\{1, \max\{0, \beta^{il}\frac{\partial v^{il}(\eta^1,\eta^2,Q^1,Q^2)}{\partial \eta^{il}_{kh\hat{l}}} - \frac{\partial c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}},\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} - \frac{\partial b^{il}_{kh\hat{l}}(\eta^{il}_{kh\hat{l}})}{\partial \eta^{il}_{kh\hat{l}}} \\ -\alpha^{il}\frac{\partial r^{il}(Q^1,Q^2,\eta^1,\eta^2)}{\partial \eta^{il}_{kh\hat{l}}}\}\}, \text{ otherwise,} \end{cases} \tag{24}$$

where $\dot{\eta}^{il}_{kh\hat{l}}$ denotes the rate of change of the relationship level $\eta^{il}_{kh\hat{l}}$. The expression (24) states that if the weighted relationship value is higher than the total marginal costs plus the total weighted marginal risk, then the level of relationship between that pair will increase. If it is lower, the relationship value will decrease. Of course, the bounds on the relationship levels must also hold.

The Dynamics of the Relationship Levels between the Retailers and the Demand Markets

The dynamics of the relationship levels between the retailers and demand markets are now described. The rate of change of such a relationship level is assumed to be equal to the difference between the weighted relationship value for the particular retailer and the sum of the associated marginal costs and weighted marginal risks, where, of course, one also must guarantee that the relationship levels do not become negative nor exceed one. Hence, one may write:

$$\dot{\eta}^{j}_{kh\hat{l}m} = \begin{cases} \gamma^{j}\frac{\partial v^{j}(Q^1,Q^3,\eta^1,\eta^3)}{\partial \eta^{j}_{kh\hat{l}m}} - \delta^{j}\frac{\partial r^{j}(Q^1,Q^3,\eta^1,\eta^3)}{\partial \eta^{j}_{kh\hat{l}m}} - \frac{\partial c^{j}_{kh\hat{l}m}(q^{j}_{kh\hat{l}m},\eta^{j}_{kh\hat{l}m})}{\partial \eta^{j}_{kh\hat{l}m}} - \frac{\partial b^{j}_{khl}(\eta^{j}_{kh\hat{l}m})}{\partial \eta^{j}_{kh\hat{l}m}}, \text{ if } 0 < \eta^{j}_{kh\hat{l}m} < 1, \\ \min\{1, \max\{0, \gamma^{j}\frac{\partial v^{j}(Q^1,Q^3,\eta^1,\eta^3)}{\partial \eta^{j}_{kh\hat{l}m}} - \delta^{j}\frac{\partial r^{j}(Q^1,Q^3,\eta^1,\eta^3)}{\partial \eta^{j}_{kh\hat{l}m}} - \frac{\partial c^{j}_{kh\hat{l}m}(q^{j}_{kh\hat{l}m},\eta^{j}_{kh\hat{l}m})}{\partial \eta^{j}_{kh\hat{l}m}} \\ -\frac{\partial b^{j}_{khl}(\eta^{j}_{kh\hat{l}m})}{\partial \eta^{j}_{kh\hat{l}m}}\}\}, \text{otherwise,} \end{cases} \tag{25}$$

where $\dot{\eta}^{j}_{kh\hat{l}m}$ denotes the rate of change of the relationship level $\eta^{j}_{kh\hat{l}m}$. Expression (25) reveals that if the weighted relationship value for the retailer with the demand market is higher than the total marginal costs plus the total weighted marginal risk, then the level of relationship between that retailer and demand market pair will increase. If it is lower, the relationship value will decrease.

The Demand Market Price Dynamics

We assume that the rate of change of the price $\rho_{3kh\hat{l}}$, denoted by $\dot{\rho}_{3kh\hat{l}}$, is equal to the difference between the demand for the product at the demand market in the currency and country and the amount of the product actually available at that particular market. Hence, if the demand for the product at the demand market at an instant in time exceeds the amount available from the various retailers and manufacturers, then the price will increase; if the amount available exceeds the demand at the price, then the price will decrease. Moreover, it is guaranteed that the prices do not become negative. Thus, the dynamics of the price $\rho_{3kh\hat{l}}$ for each $k, h, \hat{l}$ can be expressed as:

$$\dot{\rho}_{3kh\hat{l}} = \begin{cases} d_{kh\hat{l}}(\rho_3) - \sum_{j=1}^{J}\sum_{m=1}^{2} q^j_{kh\hat{l}m} - \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il}_{kh\hat{l}}, & \text{if} \quad \rho_{3kh\hat{l}} > 0, \\ \max\{0, d_{kh\hat{l}}(\rho_3) - \sum_{j=1}^{J}\sum_{m=1}^{2} q^j_{kh\hat{l}m} - \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il}_{kh\hat{l}}\}, & \text{if} \quad \rho_{3kh\hat{l}} = 0. \end{cases} \tag{26}$$

The Dynamics of the Prices at the Retailers

The prices at the retailers, whether they are physical or virtual, must reflect supply and demand conditions as well. In particular, we let $\dot{\lambda}_j$ denote the rate of change in the market clearing price associated with retailer j and we propose the following dynamic adjustment for every retailer j:

$$\dot{\lambda}_j = \begin{cases} \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} q^j_{kh\hat{l}m} - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q^{il}_{jhm}, & \text{if} \quad \lambda_j > 0, \\ \max\{0, \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L}\sum_{m=1}^{2} q^j_{kh\hat{l}m} - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H}\sum_{m=1}^{2} q^{il}_{jhm}\}, & \text{if} \quad \lambda_j = 0. \end{cases} \tag{27}$$

Hence, if the product flows from the manufacturers in the countries to a retailer exceed the amount demanded at the demand markets from a retailer, then the market-clearing price at that retailer will decrease; if, on the other hand, the volume of product flows to a retailer is less than that demanded by the consumers at the demand markets (and handled by the retailer), then the market-clearing price at that retailer will increase.

The Projected Dynamical System

We now turn to stating the complete dynamic model. In the dynamic model, the flows evolve according to the mechanisms described above; specifically, the product transactions between manufacturers and retailers evolve according to (20) and the product transactions between manufacturers and demand markets evolve according to (21) for all manufacturers. The product transactions between retailers and demand markets evolve according to (22) for all retailers, demand markets, modes, and currencies. The relationship levels between manufacturers and retailers evolve according to (23); the relationship levels between manufacturers and demand markets evolve according to (24), and the relationship levels between retailers and demand markets evolve according to (25).

Furthermore, the prices associated with the retailers evolve according to (27) for all retailers, and the demand market prices evolve according to (26).

Let X and $F(X)$ be as defined following (19) and recall the feasible set $\mathcal{K}$. Then the dynamic model described by (20)–(27) can be rewritten as a *projected dynamical system* (Nagurney and Zhang [34]) defined by the following initial value problem:

$$\dot{X} = \Pi_{\mathcal{K}}(X, -F(X)), \quad X(0) = X_0, \tag{28}$$

where $\Pi_{\mathcal{K}}$ is the projection operator of $-F(X)$ onto $\mathcal{K}$ at X and $X_0 = (Q^{10}, Q^{20}, Q^{30}, \eta^{10}, \eta^{20}, \eta^{30}, \lambda^0, \rho_3^0)$ is the initial point corresponding to the initial product transaction, relationship level, and price pattern.

The trajectory of (28) describes the dynamic evolution of the relationship levels on the social network, the product transactions on the global supply chain network, and the demand market prices and the Lagrange multipliers or shadow prices associated with the retailers. The projection operation guarantees the constraints underlying the supernetwork system are not violated. Recall that the constraint set $\mathcal{K}$ consists of the nonnegativity constraints associated with all the product transactions, the prices, as well as the relationships levels. Moreover, the relationship levels are assumed to not exceed the value of one. Hence, the projection operation is remarkably simple and, as we will see in the next Section, yields closed form expressions for computational purposes.

Following Dupuis and Nagurney [12] and Nagurney and Zhang [34], the following result is immediate due to the properties of the feasible set $\mathcal{K}$.

Theorem 2: Set of Stationary Points Coincides with Set of Equilibrium Points

The set of stationary points of the projected dynamical system (28) coincides with the set of solutions of the variational inequality problem (19); equivalently (17), and, thus, with the set of equilibrium points as defined in Definition 1.

With Theorem 2, we see that the dynamical system proposed in this Section provides the disequilibrium dynamics prior to the steady or equilibrium state of the supernetwork. Hence, once, a stationary point of the projected dynamical system is reached, that is, when $\dot{X} = 0$ in (28), that point (consisting of product transactions, relationship levels, and prices) also satisfies variational inequality (17); equivalently, (19), and is, therefore, a supernetwork equilibrium according to Definition 1.

The above described dynamics are very reasonable from an economic perspective and also illuminate that there must be cooperation between tiers of decision-makers although there may be competition within a tier.

We now state the following:

Theorem 3: Existence and Uniqueness of a Solution to the Initial Value Problem

Assume that $F(X)$ is Lipschitz continuous, that is, that

$$\|F(X') - F(X'')\| \leq \mathcal{L}\|X' - X''\|, \quad \forall X', X'' \in \mathcal{K}, \textit{ where } \mathcal{L} > 0, \tag{29}$$

Then, for any $X_0 \in \mathcal{K}$, there exists a unique solution $X_0(t)$ to the initial value problem (28).

Proof: Lipschitz continuity of the function F is sufficient for the result following Theorem 2.5 in Nagurney and Zhang [34].□

Theorem 3 gives conditions under which the trajectories associated with the initial value problem are well-defined. Note that Lipschitz continuity is not an unreasonable condition for F to satisfy in our model (see also, e.g., [29], [31], and [34]).

Under suitable conditions on the underlying functions (see also [34]), in particular, under monotonicity of the function F in (19), one can also obtain stability results for the supernetwork.

4. The Discrete-Time Algorithm

In this Section, we propose the Euler method for the computation of solutions to variational inequality (17); equivalently, the stationary points of the projected dynamical system (28). The Euler method is a special case of the general iterative scheme introduced by Dupuis and Nagurney [12] for the solution of projected dynamical systems. Besides providing a solution to variational inequality problem (17) (or (19)), this algorithm also yields a time discretization of the continuous-time adjustment process of the projected dynamical system (28). Conditions for convergence of this algorithm are given in Dupuis and Nagurney [12] and in Nagurney and Zhang [34]. In Section 5, we apply this algorithm to several numerical examples.

The Euler Method

Step 0: Initialization

Set $X^0 = (Q^{10}, Q^{20}, Q^{30}, \eta^{10}, \eta^{20}, \eta^{30}, \lambda^0, \rho_3^0) \in \mathcal{K}$. Let $\mathcal{T}$ denote an iteration counter and set $\mathcal{T} = 1$. Set the sequence $\{a_{\mathcal{T}}\}$ so that $\sum_{\mathcal{T}=1}^{\infty} a_{\mathcal{T}} = \infty$, $a_{\mathcal{T}} > 0$, $a_{\mathcal{T}} \to 0$, as $\mathcal{T} \to \infty$ (such a sequence is required for convergence of the algorithm).

Step 1: Computation

Compute $X^{\mathcal{T}} = (Q^{1\mathcal{T}}, Q^{2\mathcal{T}}, Q^{3\mathcal{T}}, \eta^{1\mathcal{T}}, \eta^{2\mathcal{T}}, \eta^{3\mathcal{T}}, \lambda^{\mathcal{T}}, \rho_3^{\mathcal{T}}) \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^{\mathcal{T}} + a_{\mathcal{T}} F(X^{\mathcal{T}-1}) - X^{\mathcal{T}-1}, X - X^{\mathcal{T}} \rangle \geq 0, \qquad \forall X \in \mathcal{K}. \tag{30}$$

Step 2: Convergence Verification

If $|X^{\mathcal{T}} - X^{\mathcal{T}-1}| \leq \epsilon$, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\mathcal{T} := \mathcal{T} + 1$, and go to Step 1.

Variational inequality subproblem (30) can be solved explicitly and in closed form. For completeness, and also to illustrate the simplicity of the proposed computational procedure in the context of the dynamic supernetwork model, we provide the explicit formulae for the computation of the vectors: $Q^{1\mathcal{T}}$, $Q^{2\mathcal{T}}$, $Q^{3\mathcal{T}}$, $\eta^{1\mathcal{T}}$, $\eta^{2\mathcal{T}}$, $\eta^{3\mathcal{T}}$, $\lambda^{\mathcal{T}}$, and $\rho_3^{\mathcal{T}}$ at iteration $\mathcal{T}$ (cf. (30)) below.

Computation of the Product Transactions

In particular, compute, at iteration $\mathcal{T}$, the $q_{jhm}^{il\mathcal{T}}$s according to:

$$q_{jhm}^{il\mathcal{T}} = \max\{0, q_{jhm}^{il\mathcal{T}-1} - a_{\mathcal{T}}(\frac{\partial f^{il}(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial q_{jhm}^{il}} + \alpha^{il} \frac{\partial r^{il}(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{2\mathcal{T}-1})}{\partial q_{jhm}^{il}}$$

$$+\delta^j \frac{\partial r^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{3\mathcal{T}-1})}{\partial q_{jhm}^{il}} + \frac{\partial c_j(Q^{1\mathcal{T}-1})}{\partial q_{jhm}^{il}} + \frac{\partial c_{jhm}^{il}(q_{jhm}^{il\mathcal{T}-1}, \eta_{jhm}^{il\mathcal{T}-1})}{\partial q_{jhm}^{il}} + \frac{\partial \hat{c}_{jhm}^{il}(q_{jhm}^{il\mathcal{T}-1}, \eta_{jhm}^{il\mathcal{T}-1})}{\partial q_{jhm}^{il}}$$

$$-\beta^{il} \frac{\partial v^{il}(\eta^{1\mathcal{T}-1}, \eta^{2\mathcal{T}-1}, Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial q_{jhm}^{il}} - \gamma^j \frac{\partial v^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{3\mathcal{T}-1})}{\partial q_{jhm}^{il}} - \lambda_j^{\mathcal{T}-1})\}, \quad \forall i, l, j, h, m; \tag{31}$$

the $q_{kh\hat{l}}^{il\mathcal{T}}$s according to:

$$q_{kh\hat{l}}^{il\mathcal{T}} = \max\{0, q_{kh\hat{l}}^{il\mathcal{T}-1} - a_{\mathcal{T}}(\frac{\partial f^{il}(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial q_{kh\hat{l}}^{il}} + \alpha^{il}\frac{\partial r^{il}(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{2\mathcal{T}-1})}{\partial q_{kh\hat{l}}^{il}}$$

$$+\frac{\partial c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il\mathcal{T}-1}, \eta_{kh\hat{l}}^{il\mathcal{T}-1})}{\partial q_{kh\hat{l}}^{il}} - \beta^{il}\frac{\partial v^{il}(\eta^{1\mathcal{T}-1}, \eta^{2\mathcal{T}-1}, Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial q_{kh\hat{l}}^{il}} + \hat{c}_{kh\hat{l}}^{il}(Q^{2\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{2\mathcal{T}-1}, \eta^{3\mathcal{T}-1})$$

$$-\rho_{3kh\hat{l}}^{\mathcal{T}-1})\}, \quad \forall i, l, k, h, \hat{l}, \tag{32}$$

and the $q_{kh\hat{l}m}^{j\mathcal{T}}$s, according to:

$$q_{kh\hat{l}m}^{j\mathcal{T}} = \max\{0, q_{kh\hat{l}m}^{j\mathcal{T}-1} - a_{\mathcal{T}}(\delta^j\frac{\partial r^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{3\mathcal{T}-1})}{\partial q_{kh\hat{l}m}^{j}} + \frac{\partial c_{kh\hat{l}m}^{j}(q_{kh\hat{l}m}^{j\mathcal{T}-1}, \eta_{kh\hat{l}m}^{j\mathcal{T}-1})}{\partial q_{kh\hat{l}m}^{j}}$$

$$+\hat{c}_{kh\hat{l}m}^{j}(Q^{2\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{2\mathcal{T}-1}, \eta^{3\mathcal{T}-1}) + \lambda_j^{\mathcal{T}-1} - \gamma^j\frac{\partial v^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{3\mathcal{T}-1})}{\partial q_{kh\hat{l}m}^{j}} - \rho_{3kh\hat{l}}^{\mathcal{T}-1})\},$$

$$\forall j, k, h, \hat{l}, m. \tag{33}$$

Computation of the Relationship Levels

At iteration $\mathcal{T}$ compute the $\eta_{jhm}^{il\mathcal{T}}$s according to:

$$\eta_{jhm}^{il\mathcal{T}} = \min\{1, \max\{0, \eta_{jhm}^{il\mathcal{T}-1} - a_{\mathcal{T}}(\frac{\partial c_{jhm}^{il}(q_{jhm}^{il\mathcal{T}-1}, \eta_{jhm}^{il\mathcal{T}-1})}{\partial \eta_{jhm}^{il}} + \frac{\partial \hat{c}_{jhm}^{il}(q_{jhm}^{il\mathcal{T}-1}, \eta_{jhm}^{il\mathcal{T}-1})}{\partial \eta_{jhm}^{il}} + \frac{\partial b_{jhm}^{il}(\eta_{jhm}^{il\mathcal{T}-1})}{\partial \eta_{jhm}^{il}}$$

$$+\frac{\partial \hat{b}_{jhm}^{il}(\eta_{jhm}^{il\mathcal{T}-1})}{\partial \eta_{jhm}^{il}} + \alpha^{il}\frac{\partial r^{il}(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{2\mathcal{T}-1})}{\partial \eta_{jhm}^{il}} + \delta^j\frac{\partial \hat{r}^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{3\mathcal{T}-1})}{\partial \eta_{jhm}^{il}}$$

$$-\beta^{il}\frac{\partial v^{il}(\eta^{1\mathcal{T}-1}, \eta^{2\mathcal{T}-1}, Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial \eta_{jhm}^{il}} - \gamma^j\frac{\partial v^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{3\mathcal{T}-1})}{\partial \eta_{jhm}^{il}})\}\}, \quad \forall i, l, j, h, m. \tag{34}$$

Furthermore, at iteration $\mathcal{T}$ compute the $\eta_{kh\hat{l}}^{il\mathcal{T}}$s according to:

$$\eta_{kh\hat{l}}^{il\mathcal{T}} = \min\{1, \max\{0, \eta_{kh\hat{l}}^{il\mathcal{T}-1} - a_{\mathcal{T}}(\frac{\partial c_{kh\hat{l}}^{il}(q_{kh\hat{l}}^{il\mathcal{T}-1}, \eta_{kh\hat{l}}^{il\mathcal{T}-1})}{\partial \eta_{kh\hat{l}}^{il}} + \alpha^{il}\frac{\partial r^{il}(Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{2\mathcal{T}-1})}{\partial \eta_{kh\hat{l}}^{il}}$$

$$+\frac{\partial b_{kh\hat{l}}^{il}(\eta_{kh\hat{l}}^{il\mathcal{T}-1})}{\partial \eta_{kh\hat{l}}^{il}} - \beta^{il}\frac{\partial v^{il}(\eta^{1\mathcal{T}-1}, \eta^{2\mathcal{T}-1}, Q^{1\mathcal{T}-1}, Q^{2\mathcal{T}-1})}{\partial \eta_{kh\hat{l}}^{il}})\}\}, \quad \forall i, l, k, h, \hat{l}. \tag{35}$$

At iteration $\mathcal{T}$ compute the $\eta^{j\mathcal{T}}_{kh\hat{l}m}$s according to:

$$\eta^{j\mathcal{T}}_{kh\hat{l}m} = \min\{1, \max\{0, \eta^{j\mathcal{T}}_{kh\hat{l}m} - a_{\mathcal{T}}(\delta^j \frac{\partial r^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{3\mathcal{T}-1})}{\partial \eta^j_{kh\hat{l}m}} + \frac{\partial c^j_{kh\hat{l}m}(q^{j\mathcal{T}-1}_{kh\hat{l}m}, \eta^{j\mathcal{T}-1}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}}$$

$$+ \frac{\partial b^j_{khl}(\eta^{j\mathcal{T}-1}_{kh\hat{l}m})}{\partial \eta^j_{kh\hat{l}m}} - \gamma^j \frac{\partial v^j(Q^{1\mathcal{T}-1}, Q^{3\mathcal{T}-1}, \eta^{1\mathcal{T}-1}, \eta^{3\mathcal{T}-1})}{\partial \eta^j_{kh\hat{l}m}})\}\}, \quad \forall j, k, h, \hat{l}, m. \tag{36}$$

Computation of the Shadow Prices

At iteration $\mathcal{T}$, compute the $\lambda^{\mathcal{T}}_j$s according to:

$$\lambda^{\mathcal{T}}_j = \max\{0, \lambda^{\mathcal{T}-1}_j - a_{\mathcal{T}}(\sum_{m=1}^{2} \left[\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{h=1}^{H} q^{il\mathcal{T}-1}_{jhm} - \sum_{k=1}^{K}\sum_{h=1}^{H}\sum_{\hat{l}=1}^{L} q^{j\mathcal{T}-1}_{kh\hat{l}m}\right])\}, \quad \forall j, \tag{37}$$

Computation of the Demand Market Prices

Finally, at iteration $\mathcal{T}$ compute the demand market prices, the $\rho^{\mathcal{T}}_{3kh\hat{l}}$s, according to:

$$\rho^{\mathcal{T}}_{3kh\hat{l}} = \max\{0, \rho^{\mathcal{T}-1}_{3kh\hat{l}} - a_{\mathcal{T}}(\sum_{j=1}^{J}\sum_{m=1}^{2} q^{j\mathcal{T}-1}_{kh\hat{l}m} + \sum_{i=1}^{I}\sum_{l=1}^{L} q^{il\mathcal{T}-1}_{kh\hat{l}} - d_{kh\hat{l}}(\rho^{\mathcal{T}-1}_3))\}, \quad \forall k, h, \hat{l}. \tag{38}$$

As one can see from the above expressions, the algorithm is initialized with a vector of product transactions, relationship levels, and prices. For example, the relationship levels may be set to zero (and the same initialization may be done for the prices and the product transactions). The product transactions, shadow prices, and the demand market prices are computed in the global supply chain network of the supernetwork. In particular, the product transactions between manufacturers and retailers are computed according to (31), the product transactions between manufacturers and demand markets are computed according to (32), and the product transactions between retailers and demand markets are computed according to (33). The relationship levels are computed in the social network component of the supernetwork according to (34), (35), and (36), respectively. Finally, the shadow prices are computed according to (37) and the demand market prices are computed according to (38).

The dynamic supernetwork system will then evolve according to the discrete-time adjustment processes (31) through (38) until a stationary/equilibrium point of the projected dynamical system

(28) (equivalently, and a solution to variational inequality (17)) is achieved. Once the convergence tolerance has been reached then the equilibrium conditions according to Definition 1 are satisfied as one can see from (31) through (38).

5. Numerical Examples

In this Section, we applied the Euler method described in the preceding section to several supernetwork numerical examples. The algorithm was implemented in FORTRAN and the computer system used was a SUN system located at the University of Massachusetts at Amherst.

The convergence criterion utilized was that the absolute value of the global product transactions, relationship levels, and prices between two successive iterations differed by no more than 10^{-4}. For the examples, the sequence $\{a_{\mathcal{T}}\}$ was set to $10\{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \ldots\}$ in the algorithm. Such a sequence satisfies the conditions required for convergence of the algorithm.

We initialized the Euler method as follows: all the initial global product transactions, relationship levels, and prices were set to zero. We assumed that the risk was represented through variance-covariance matrices for the manufacturers as well as for the retailers with the resulting risk functions being quadratic (see also [30]) and dependent only on the product transactions.

Detailed descriptions of the specific data for the examples are given below.

Example 1

The first numerical example consisted of two countries with two manufacturers in each country; two currencies, two retailers, two demand markets, with only physical transactions between manufacturers and retailers and the retailers and demand markets. Electronic transactions were allowed between manufacturers and the demand markets. Hence, $L = 2$, $I = 2$, $H = 2$, $J = 2$, $K = 2$ with $m = 1$. This yielded a numerical example in which (cf. Figure 1) there were four top tier nodes in both the global supply chain network and the social network components of the supernetwork and two middle tier nodes and eight bottom tier nodes in each network component. There were two links (representing each currency) from each top-tiered node to each middle-tiered node in both the global supply chain network and the social network. There was a single link joining each of the two middle-tiered nodes with each bottom-tiered node in each network. Finally, there was a link joining each top-tiered node to each bottom-tiered node in both the supply chain network and the social network to represent B2C commerce through electronic transactions between the manufacturers and the demand markets with the supply chain handling the product transactions and the social network – the relationship levels.

The data for the first example were constructed for easy interpretation purposes. We set the variance-covariance matrices associated with the risk functions to the identity matrices. In addition, for the sake of simplicity, we considered the possibility of the existence of relationship levels only between the manufacturers and the retailers, and between the retailers and the demand markets. Please refer to Tables 1 to 5 for a compact exposition of the notation.

The transaction cost functions faced by the manufacturers associated with transacting with the retailers were given by:

$$c^{il}_{jhm}(q^{il}_{jhm}, \eta^{il}_{jhm}) = .5(q^{il}_{jhm})^2 + 3.5q^{il}_{jhm} - \eta^{il}_{jhm}, \quad \text{for } i = 1, 2; l = 1; j = 1, 2; h = 1, 2; m = 1.$$

The production cost functions faced by the manufacturers were

$$f^{il}(Q^1, Q^2) = .5(\sum_{j=1}^{2}\sum_{h=1}^{2} q^{il}_{jh1})^2, \quad \text{for } i = 1, 2; l = 1, 2.$$

The handling costs of the retailers were given by:

$$c_j(Q^1) = .5(\sum_{i=1}^{2}\sum_{h=1}^{2} q^{i1}_{jh1})^2, \quad \text{for } j = 1, 2.$$

The transaction costs of the retailers associated with transacting with the manufacturers in the two countries were given by:

$$\hat{c}^{il}_{jhm}(q^{il}_{jhm}, \eta^{il}_{jhm}) = 1.5q^{il2}_{jhm} + 3q^{il}_{jhm}, \quad \text{for } i = 1, 2; l = 1, 2; j = 1, 2; h = 1, 2; m = 1.$$

The demand functions at the demand markets were:

$$d_{111}(\rho_3) = -2\rho_{3111} - 1.5\rho_{3121} + 1000, \quad d_{121}(\rho_3) = -2\rho_{3121} - 1.5\rho_{3111} + 1000,$$

$$d_{211}(\rho_3) = -2\rho_{3211} - 1.5\rho_{3221} + 1000, \quad d_{221}(\rho_3) = -2\rho_{3221} - 1.5\rho_{3211} + 1000,$$

$$d_{112}(\rho_3) = -2\rho_{3112} - 1.5\rho_{3122} + 1000, \quad d_{122}(\rho_3) = -2\rho_{3122} - 1.5\rho_{3112} + 1000,$$

$$d_{212}(\rho_3) = -2\rho_{3212} - 1.5\rho_{3222} + 1000, \quad d_{222}(\rho_3) = -2\rho_{3222} - 1.5\rho_{3212} + 1000,$$

and the transaction costs between the retailers and the consumers at the demand markets were given by:

$$\hat{c}^j_{kh\hat{l}m}(Q^2, Q^3, \eta^2, \eta^3) = q^j_{khlm} - \eta^j_{kh\hat{l}m} + 5, \quad \text{for } j = 1,2; k = 1,2; h = 1,2; l = 1,2; m = 1.$$

The transaction cost functions associated with the electronic transactions between the manufacturers in the two countries and the demand markets were:

$$c^{il}_{kh\hat{l}}(q^{il}_{kh\hat{l}}, \eta^{il}_{kh\hat{l}}) = .5(q^{il}_{kh\hat{l}})^2 + q^{il}_{kh\hat{l}}, \quad \text{for } i = 1,2; l = 1,2; k = 1,2; h = 1,2; \hat{l} = 1,2.$$

The transaction costs associated with the electronic transactions from the perspective of the consumers at the demand markets were as follows:

$$\hat{c}^{il}_{kh\hat{l}}(Q^2, Q^3, \eta^2, \eta^3) = .1q^{il}_{kh\hat{l}} + 1, \quad \text{for } i = 1,2; l = 1,2; k = 1,2; h = 1,2; \hat{l} = 1,2.$$

The relationship value functions were as follows:

$$v^{il}(\eta^1, \eta^2, Q^1, Q^2) = \eta^{il}_{jhm}, \quad \text{for } i = 1,2; l = 1,2; j = 1,2; h = 1,2; m = 1;$$

$$v^j(\eta^1, \eta^3, Q^1, Q^3) = \eta^j_{kh\hat{l}m}, \quad \text{for } j = 1,2; k = 1,2; h = 1,2; \hat{l} = 1,2; m = 1.$$

The relationship cost functions were:

$$b^{il}_{jhm}(\eta^{il}_{jhm}) = 2\eta^{il}_{jhm}, \quad \text{for } i = 1,2; l = 1,2; j = 1,2; h = 1,2; m = 1,$$

$$b^j_{kh\hat{l}m}(\eta^j_{kh\hat{l}m}) = \eta^j_{kh\hat{l}m}, \quad \text{for } j = 1,2; k = 1,2; h = 1,2; \hat{l} = 1,2; m = 1.$$

All other functions were set equal to zero.

For the first example we assumed that all the weights associated with the different criteria were set equal to one by all the decision-makers. Hence, in this example, the manufacturers and the retailers assigned the same weight to profit maximization, risk minimization, and relationship value maximization.

The Euler method converged and yielded the following equilibrium product transaction pattern:

$$q_{jh1}^{il*} = 2.160, \quad \forall i, l, j, h, \quad q_{kh\hat{l}}^{j*} = 2.160, \quad \forall j, k, h, \hat{l},$$

with the equilibrium volumes of product transacted electronically between manufacturers and the demand markets being:

$$q_{kh\hat{l}}^{il*} = 22.376, \quad \forall i, l, k, h, \hat{l}.$$

Clearly, in this example, consumers preferred to conduct their transactions directly with the manufacturers in an electronic manner.

The vector λ^* had components: $\lambda_1^* = \lambda_2^* = 224.322$, and the computed demand prices at the demand markets were: $\rho_{3111}^* = \rho_{3121}^* = \rho_{3211}^* = \rho_{3221}^* = \rho_{3112}^* = \rho_{3122}^* = \rho_{3212}^* = \rho_{3222}^* = 258.917$.

The equilibrium relationship levels were all equal to zero.

Example 2

Example 2 was constructed from the preceding example as follows. We kept the data as in Example 3 except that we now increased the weight associated with relationship value maximization for both manufacturers and retailers from one to twenty.

The Euler method again converged yielding the same equilibrium pattern as in Example 1 except that now all the equilibrium relationship levels were equal to 1 for the relationships between the manufacturers and the retailers, that is, they were at their upper bounds. The other relationship levels remained at level zero.

Example 3

Example 3 was constructed from Example 2 in the following manner. The data were identical to that in Example 2 except that now we modified the demand function associated with demand market 1, currency 1, and country 1 so that the intercept of the demand function (see Example 1) increased by 100.

The Euler method yielded the following equilibrium product transaction pattern:

$$q_{jh1}^{il*} = 2.192, \quad \forall i, l, j, h,$$

$$q^{1*}_{111} = 4.631,\ q^{1*}_{121} = 0.861,\ q^{1*}_{211} = 2.007,\ q^{1*}_{221} = 2.007,$$

$$q^{1*}_{112} = 2.007,\ q^{1*}_{122} = 2.007,\ q^{1*}_{212} = 2.007,\ q^{1*}_{222} = 2.007,$$

$$q^{2*}_{111} = 4.631,\ q^{2*}_{121} = 0.861,\ q^{2*}_{211} = 2.007,\ q^{2*}_{221} = 2.007,$$

$$q^{2*}_{112} = 2.007,\ q^{2*}_{122} = 2.007,\ q^{2*}_{212} = 2.007,\ q^{2*}_{222} = 2.007.$$

The amounts of product transacted electronically between manufacturers and the demand markets remained at: $q^{il*}_{kh\hat{l}} = 22.376$, for all $i, l, k, h, \hat{l}$.

The vector λ^* had components: $\lambda^*_1 = \lambda^*_2 = 227.126$, and the computed demand prices at the demand markets were: $\rho^*_{3111} = 295.570; \rho^*_{3121} = 243.925; \rho^*_{3211} = 259.619; \rho^*_{3221} = 259.619;$ $\rho^*_{3112} = \rho^*_{3122} = \rho^*_{3212} = \rho^*_{3222} = 259.619$.

The computed equilibrium relationship levels were equal to one for the manufacturer/retailer combinations and zero, otherwise.

These examples (although stylized) have been presented to show both the model and the computational procedure. Obviously, different input data and dimensions of the problems solved will affect the equilibrium product transaction, relationship level, and price patterns. One now has a powerful financial engineering tool with which to explore the effects of perturbations to the data as well as the effects of changes in the number of manufacturers, retailers, countries, currencies, and/or demand markets.

6. Summary and Conclusions

In this paper, we developed a supernetwork model (in both static and dynamic forms) that integrated global supply chain networks, which allowed for physical as well as electronic transactions, with social networks, in which relationship levels were made explicit. Both networks had three tiers of decision-makers, consisting of: manufacturers, retailers, as well as consumers associated with the demand markets. We allowed for physical as well as electronic transactions between the decision-makers in the supernetwork. The relationship levels could affect not only the risk functions but also the transaction cost functions and did have associated costs. Moreover, we considered multicriteria decision-making behavior in which the manufacturers who could be located in different countries as well as the retailers, who since they could be virtual needed not to be country-specific, were permitted to weight, in an individual manner, their objective functions of profit maximization, risk minimization, and relationship value maximization.

We first modeled the supernetwork in equilibrium, in which the product transactions between the tiers as well as the relationship levels coincide and established the variational inequality formulation of the governing equilibrium conditions. We then proposed the underlying (disequilibrium) dynamics associated with the continuous-time adjustment process(es) and constructed the projected dynamical system formulation. We proved that the set of stationary points of the projected dynamical system coincides with the set of solutions of the variational inequality problem. We also provided conditions under which the dynamic trajectories of the global product transactions, relationship levels, and prices are well-defined. We proposed a discrete-time algorithm to approximate the continuous-time adjustment processes and applied it to several simple numerical examples for completeness and illustrative purposes.

This framework generalizes the recent work of Wakolbinger and Nagurney [42] in the integration of social networks and supply chains with electronic commerce to the global dimension and introduces more general risk functions, transaction cost functions, and relationship value functions that had been utilized therein.

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